

May 2025
M.Sc.
Second Semester
CORE – 05
MATHEMATICS
Course Code: MMAC 2.11
(Numerical Analysis)

Total Mark: 70

Pass Mark: 28

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT-I

1. (a) Write a short note on error in numerical analysis. 3
(b) Determine the root of $\tan x = x$ using secant method. 5
(c) Perform four iterations of the Newton-Raphson method to obtain the root of the equation $f(x) \equiv \sin x = 1 + x^3$ which lies in the interval $(-2, -1)$. 6
2. (a) What is an iterative method? What are the criteria for termination of iterative methods? 2
(b) Perform three iterations of the Chebyshev's method to find the approximate value of $\frac{1}{7}$. (Take $x_0 = 0.1$) 6
(c) Perform three iterations of the multipoint method to find the smallest positive root of the equation $f(x) \equiv x^3 - 5x + 1 = 0$. 6

UNIT-II

3. (a) Solve the equations 6
$$x + y + z = 6$$
$$2x + (3 + \varepsilon)y + 4z = 20$$
$$2x + y + 3z = 13$$
using Gauss elimination method, where ε is small such that $1 \pm \varepsilon \approx 1$.

- (b) Solve the system of equations by LU decomposition method taking $u_{ii} = 2$ for $i = 1, 2, 3$ 8

$$x + y - z = 2$$

$$2x + 3y + 5z = -3$$

$$3x + 2y - 3z = 6$$

4. (a) Find the inverse of the matrix $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ by partition method. 7

- (b) Find the largest eigenvalue and its corresponding eigenvector of

the matrix $\begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{bmatrix}$ by performing four iterations of power

method. 7

UNIT-III

5. (a) Deduce the Lagrange interpolating polynomial for the data. 5

x	0	3	4	5	7	10
$f(x)$	3	31	69	131	351	1011

- (b) Construct the interpolating polynomial that fits the data

x	0.0	0.1	0.2	0.3	0.4	0.5
$f(x)$	-1.5	-1.27	-0.98	-0.63	-0.22	0.25

using Newton's backward interpolation and hence estimate $f(0.45)$. 5

- (c) Prove the following: 1×3=3

(i) $\mu^2 = 1 + \frac{1}{4}\delta^2$

(ii) $\Delta(f(x)/g(x)) = (g(x)\Delta f(x) - f(x)\Delta g(x)) / g(x)g(x+h)$

(iii) $(1+\Delta)(1-\nabla) = 1$

- (d) Compute $\Delta^3(1-2x)(1-3x)(1-4x)$. 1

6. (a) Construct the Hermite interpolation polynomial that fits the data

x	$f(x)$	$f'(x)$
1	7.389	14.778
2	54.598	109.196

Estimate the value of $f(x)$ at $x = 1.5$. If the data represents the function $f(x) = e^{2x}$, find the absolute error at $x = 1.5$.

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- (b) For the following data, obtain the piecewise quadratic interpolating polynomials

x	-2	0	1	3	4
$f(x)$	-23	1	4	82	193

Interpolate at $x = -0.5$ and $x = 2.0$.

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UNIT-IV

7. (a) Find the values of $f'(9)$ and $f''(9)$ for the given data
 $f(6) = 1.556, f(7) = 1.690, f(9) = 1.908, f(12) = 2.158$.

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- (b) The function $y = \sin x$ is tabulated below

x	0.7	0.8	0.9	1.0	1.1	1.2	1.3
y	0.644	0.717	0.783	0.841	0.891	0.932	0.963
	218	356	327	471	207	039	558

Find the derivative at the point $x = 1.3$ and compute its actual error.

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- (c) Find $f''(5)$ using divided difference formula for the data
 $f(3) = -13, f(5) = 23, f(11) = 899, f(27) = 17315, f(34) = 35606$.

6

8. (a) Find the value of $\int_0^5 \frac{dx}{4x+5}$ using Simpson's 1/3 rule and hence find the value of $\log_e 5$. (Take $n = 10$)

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- (b) Find the value of $I = \int_1^2 \frac{dx}{3x+5}$ using Gauss two- and three-point rules. Find the absolute errors in the solutions.

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UNIT-V

9. (a) Reduce the system of higher order initial value problem into a system of first order differential equations. 4
- $$y''' = e^x + y' + u' + u + y, y(1) = 3, y'(1) = 1$$
- $$u'' = e^x + 9xu + 6u' + 9y' + 10y, u(1) = 1, u'(1) = 2$$
- (b) Find $y(0.2)$ using mid-point method, if $\frac{dy}{dx} = 2x + 3y$, $y(0) = 1$ with $h = 0.1$. 5
- (c) Use Picard's method to solve $\frac{dy}{dx} = 1 + xy$, with $x_0 = 2$, $y_0 = 0$. 5
10. (a) Solve the system of equations
- $$u' = -3u + 2v, u(0) = 0$$
- $$v' = 3u - 4v, v(0) = 0.5$$
- with $h = 0.2$, using the classical Runge-Kutta fourth order method. 6
- (b) Find the solution at $t = 0.5$ for the initial value problem
- $$y' = t - y^2, y(0) = 1$$
- by the Adam-Bashforth method of order three with $h = 0.1$. 8
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