2024

M.Sc.

Second Semester

CORE – 05

PHYSICS

Course Code: MPHC 2.11

(Electrodynamics)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Obtain the Laplace's equation in cartesian coordinates and find its solution. 6

(b) Consider two concentric spherical shell of radius r_a and r_b and potential V_a and V_b at their surfaces. Find the potential between them using the Laplace equation. 5

(c) A homogeneous dielectric $(\varepsilon_r = 2.5)$ fills region $1(x \leq 0)$ while region $2(x \geq 0)$ is free space. If $\vec{D}_1 = 12\hat{a}_x - 10\hat{a}_y + 4\hat{a}_z \text{ nCm}^{-2}$. Find $\vec{D}_2$ and θ_2. 3

2. (a) Show the Coulomb gauge condition using Maxwell's field equations in terms of scalar and vector potential. 6

(b) A dielectric sphere with radius a and dielectric constant ε is placed in a uniform electric field E along the Z-axis in vacuum. Use the spherical coordinate system with the origin at the centre of the sphere. What is the potential ϕ in vacuum $(r > a)$ and the potential ϕ in the dielectric sphere $(z < a)$. 5

(c) A charged long cylinder of radius a has a volume charge density, $r = kr$, where k is a constant and r is the distance from the axis of the cylinder. Show that the electric field is given by

$$\vec{E} = \frac{kr^2}{3\varepsilon}\hat{a}_r \qquad r < a$$ 3

UNIT–II

3. (a) Discuss the reflection and refraction at the boundary of two dielectric media for a normal incidence. Show that the reflectance and transmittance is equal to unity. 8

   (b) The electric field intensity of an electromagnetic wave in free space is given by $E_y = 0, E_z = 0, E_x = E_0 \cos\omega\left(t - \frac{z}{v}\right)$. Determine the expression for the components of the magnetic field intensity $\vec{H}$. Also, find $\frac{E_x}{E_y}$. 6

4. (a) Derive the propagation of plane electromagnetic waves in an anisotropic non-conducting medium. 8

   (b) An electromagnetic wave propagates in a non-magnetic medium with relative permittivity $\in = 4$. The magnetic field for this wave is $\vec{H}(x, y) = \hat{k}H_0 \cos\left(\omega t - ax - a\sqrt{3}y\right)$. Where H_0 is a constant. Find the corresponding electric field $\vec{E}(x, y)$. 3

   (c) A circular wave guide has an internal radius of 2.5 cm. Calculate the cut-off wavelength, the guide wavelength and the wave impedance of the guide when operated at a frequency of 8 GHz and propagating in TE_{11} mode. 3

UNIT–III

5. (a) Derive the expression for an electromagnetic wave travelling between parallel conducting planes. 7

   (b) A metallic wave guide of square cross-section of side *L* is excited by an electromagnetic wave of wave-number *k*. Find the group velocity of the TE_{11} mode. 4

(c) Show that the function $F = E^{-\alpha z}\left\{\frac{\omega}{v}(x - vt)\right\}$ satisfies the wave equation $\nabla^2 F = \left(\frac{1}{c^2}\right)\ddot{F}$ provided that the wave velocity is given by

$$v = c\left(1 + \frac{\alpha^2 c^2}{\omega^2}\right)^{1/2}.$$ 3

6. (a) Explain and derive the expression for TE and TM mode for a circular waveguide. 10

(b) A lossless air-filled rectangular waveguide has dimensions $a = 7.214$ cm and $b = 3.404$ cm. For the dominant mode propagation at 2.6 GHz, the guide transports 200 W of average power. Find the level of excitation of the *E* field. 4

UNIT–IV

7. (a) Derive the Lienard- Wiechert potential and field for a point charge.

Show that $\phi = \frac{q}{4\pi\varepsilon_0}\frac{1}{R - \vec{\beta}\cdot\vec{R}}\int_{-\infty}^{+\infty}\delta(t'')dt''$. 10

(b) A piece of wire bent into a loop, as shown in Figure, carries a current that increases linearly with time $I(t) = kt\,(-\infty < t < \infty)$. Calculate the retarded vector potential *A* at the center. Find the electric field at the center. 4

8. (a) Starting from the radiation magnetic field, $\vec{B}_r = \frac{\vec{r}\times\vec{E}_r}{rc}$ and radiation electric field $\vec{E}_t = \frac{q}{4\pi\varepsilon_0}\frac{\vec{r}\times(\vec{r} - \vec{r}\vec{\beta})\times\vec{\beta}'}{(r - \vec{r}\cdot\vec{\beta})^3 c}$. Derive the Larmor formula of radiation from accelerated charge. 8

(b) An oscillating point dipole of moment $\vec{p}(t) = \hat{z}p_0 \cos \omega t$, generates time-dependent electric and magnetic fields. At a distance *r* far away from the dipole, the vector potential due to this dipole, in SI units is $\vec{A} = \hat{z}\frac{\mu_0 p_0 \omega}{4\pi r}\sin \omega\left(t - \frac{r}{c}\right)$. Find the total power radiated from this dipole. 6

UNIT–V

9. (a) Deduce the expression for electromagnetic field tensor and explain its importance. 10

(b) Show that the wave equation $\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = 0$ is covariant with respect to the Lorentz transformations.

$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma\left(t' + \frac{v}{c^2}x''\right)$$

4

10. (a) Using the equation of continuity, $\vec{\nabla}.J + \frac{\partial \rho}{\partial t} = 0$. Express ρ and J as the component of a four-vector J_α. Show that it is invariant. 7

(b) Explain Minkowski's four-dimensional space time, time-like, and space-like intervals. 7
