2024
M.Sc.
Fourth Semester
DISCIPLINE SPECIFIC ELECTIVE – 04
MATHEMATICS
Course Code: MMAD 4.21
(Fluid Mechanics)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Describe the Lagrange's and Eulerian methods of describing the fluid flows. 3+3=6
   (b) Determine the acceleration at the point (2, 1, 3) at $t = 0.5$ sec, if $u = yz + t$; $v = xz - t$ and $w = xy$. 3
   (c) Show that $(x^2/a^2)\tan^2 t + (y^2/b^2)\cot^2 t = 1$ is a possible form of boundary surface of a liquid. 5

2. (a) Test whether the motion specified by

$$\vec{q} = \frac{k^2(x\hat{j} - y\hat{i})}{x^2 + y^2}, (\kappa = \text{constant})$$ is a possible motion for an

   incompressible fluid. If so, determine the equation of streamlines. Also, test whether the motion is of the potential kind and if so, determine the velocity potential. 2+2+1+2=7
   (b) Derive Euler's equation of motion for inviscid fluid in vector form and hence generate Lamb's hydrodynamical equations. 5+2=7

UNIT–II

3. (a) Verify that the curves of constant velocity potential and constant stream functions cut orthogonally at their points of intersections. 3
   (b) Find the image of a doublet with regard to a circle. 5

(c) Two sources, each of strength *m* are placed at the points $(-a, 0), (a, 0)$ and a sink of strength $2m$ at the origin. Show that the strength are curves $(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda xy)$, where λ is a variable parameter. Also, show that the fluid speed at any point is $\frac{2ma^2}{r_1 r_2 r_3}$ where r_1, r_2 and r_3 are the distances of the points from the sources and the sink. 4+2=6

4. (a) If the expression for stream function is described by $\psi = x^3 - 3xy^2$ determine whether flow is rotational or irrotational. If the flow is irrotational then indicate the correct value of the velocity potential. 5

(b) If $\phi = 3xy$, find *x* and *y* components of velocity at (1, 3) and (3, 3). Determine the discharge passing between streamlines passing through these points. 1+2=3

(c) State and prove the Milne-Thomson circle theorem. 6

UNIT–III

5. (a) Derive complex potential due to a rectilinear vortex filament. 7

(b) Prove that for the complex potential $\tan^{-1} z$ the streamlines and equipotentials are circles. Find the velocity at any point and examine the singularities at $z = \pm i$, 5+2=7

6. (a) Determine the stream function when the strength of the vortex filaments are equal. 6

(b) An infinite single row of equidistant rectilinear parallel vortices of same strength *k* is at a distance *a* apart. Obtain complex potential, velocity components and streamlines. 4+2+2=8

UNIT–IV

7. (a) Discuss plane Poiseuille flow. 7

(b) The stress tensor at a point is given with respect to the axes x, y, z

by the values $\sigma_{ij} = \begin{pmatrix} 6 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$.

Determine the principal stress values and principal stress directions represented by the axes x', y' and z'. 7

8. (a) Discuss Hagen-Poiseuille flow. 7

(b) Explain the significance of Reynolds number. 3

(c) Velocity field at a point is given by

$1+2y-3z, 4-2x+5z, 6+3x-5y$. Show that it represents a rigid body motion. 4

UNIT–V

9. (a) Discuss the elementary analysis of normal shock wave in brief. 7

(b) Derive the equation of motion of a gas in an isentropic flow process. 7

10. (a) Define nozzle. Discuss the flow of a gas through a nozzle. 1+6=7

(b) In a duct in which air is flowing, a normal shock wave occurs at a Mach number 1.5. The static pressure and temperature upstream of the shock are 170 kN/m^2 and 23°C respectively. Assuming $\gamma = 1.5$, calculate pressure, temperature and Mach number downstream of the shock. 3+2+2=7