2024

M.Sc.

Fourth Semester

DISCIPLINE SPECIFIC ELECTIVE – 03

MATHEMATICS

Course Code: MMAD 4.11 (B)

(Discrete Mathematics)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Let $n > 1$ be a natural number. How many elements are in the set $\{(a,b) \in \mathbb{N} \times \mathbb{N} \mid a \le b \le n\}$. Explain. 6

(b) Prove that the intervals $]0,1[$ and $]8,10[$ have the same cardinality. 4

(c) Using mathematical induction, prove that $2^n > n^2$, for $n > 5$. 4

2. (a) Let *A*, *B*, and *C* be sets. 5×2=10

(i) Find a counterexample to the statement

$A \cup (B \cap C) = (A \cup B) \cap C$

(ii) Without using Venn diagrams, prove that

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) Using mathematical induction, prove that $8^n - 3^n$ is divisible by 5 for all $n \ge 1$ 4

UNIT–II

3. (a) Using truth tables, show that 4×2=8

(i) $q \to (p \to q)$ is a tautology.

(ii) $[p \wedge q] \wedge [(\neg p) \vee (\neg q)]$ is a contradiction.

(b) Show that the following argument is valid: 6

$$\begin{array}{c} p \to \neg q \\ r \to q \\ r \\ \hline \neg p \end{array}$$

4. (a) Express $p \to (q \wedge r)$ in disjuctive normal form. 7

(b) Determine the validity of the following argument: 7

If I like maths, then I will study

Either I don't study or I pass maths

If I don't graduate, then I didn't pass maths

If I graduate, then I studied

UNIT–III

5. (a) Find the conjunctive normal form of the Boolean expression

$f(x, y, z) = xy'z + xyz$ 7

(b) Using a Karnaugh map for three variables, minimise the Boolean expression $x'y'z + xz + xy'z'$. 7

6. (a) Find the Boolean expression that defines the function f by

$f(0,0,0) = f(0,0,1) = f(1,1,0) = f(0,1,1) = 0$

$f(1,0,0) = f(0,1,0) = f(1,0,1) = f(1,1,1) = 1$ 7

(b) Using a Karnaugh map for three variables, minimise the Boolean expression $xz + yz' + y'z'$. 7

UNIT–IV

7. Let $A = \{1, 2, 4, 6, 8,\}$ and $\forall a, b \in A$ define $aRb \Leftrightarrow \frac{b}{a} \in \mathbb{Z}$. Prove that R defines a partial order on A. Draw its Hasse diagram. 6

8. (a) Draw the Hasse diagram for the partial order
$(\{a\},\{a,b\},\{a,b,c\},\{a,b,c,d\},\{a,c\},\{c,d\},\subseteq)$.
List all minimal, minimum, maximal, and maximum elements of the partial order. 6

(b) What is a totally ordered set? Show that any totally ordered set is a lattice. 2

UNIT–V

9. (a) For a finite number of finite sets $A_1, A_2, \ldots, A_n$, find the number of elements in the union $A_1 \cup A_2 \cup \ldots \cup A_n$. 9

(b) Of any five points chosen within an equilateral triangle whose sides have length 1, show that two are within a distance of $\frac{1}{2}$ of each other. 5

10. (a) In any list of *n* natural numbers, prove that there must always exist a string of consecutive numbers whose sum is divisible by *n*. 5

(b) In a group of 82 students, 59 are taking English, 46 are taking mathematics, and 12 are taking neither of these subjects. How many are taking both English and mathematics? 4

(c) In a 12-day period, a small business mailed 195 bills to customers. Show that during some period of three consecutive days, at least 49 bills were mailed. 5
