2024

M.Sc.

Fourth Semester

CORE – 11

MATHEMATICS

Course Code: MMAC 4.11

(Mathematical Methods)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) State and prove the second translation or shifting property of Laplace transform. 4

(b) If $F(t)=\begin{cases}\sin t, & 0<t<\pi \\ 0, & t>\pi\end{cases}$, show that $\mathcal{L}\{F(t)\}=\dfrac{1+e^{-\pi s}}{s^2+1}$. 3

(c) Find the inverse Laplace transform of $F(s)=\dfrac{2s+3}{s^2+4s+13}$. 4

(d) Find the Fourier sine and cosine transform of $f(x)=\begin{cases}1, & 0<x<a \\ 0, & x>a\end{cases}$. 3

2. (a) Evaluate the integral $\int_0^\infty \dfrac{dx}{\left(a^2+t^2\right)\left(b^2+t^2\right)}$ using Parseval's identity. 4

(b) Solve the simultaneous differential equation using Laplace transform $\left(D-2\right)x-\left(D-2\right)y=\sin t, (D^2+1)x+2Dy=0$, if $x(0)=x'(0)=y(0)=0$. 5

(c) Find the bounded solution of $\dfrac{\partial^2 y}{\partial x^2}-\dfrac{\partial^2 y}{\partial t^2}=xt$, subject to the boundary condition $y(x, 0)=0=y_t(x, 0), y(0, t)=0$. 5

UNIT–II

3. (a) For what value of λ the function $\varphi(x) = 1 + \lambda x$ is a solution of the integral equation $x = \int_0^x e^{x-t}\varphi(t)dt$. 3

(b) Form integral equation corresponding to the differential equation $Y'' + \lambda Y = 0$ with the initial conditions $Y(0) = 0, Y(L) = 0$. 5

(c) Solve the integral equation $\varphi(x) = 1 + \int_0^x xt\varphi(t)dt$ by successive substitution method. 6

4. (a) Using the method of successive approximation, solve the integral equation $\varphi(x) = 2x + 2 - \int_0^x \varphi(t)dt$ by taking $\varphi_0(x) = 1$ and $\varphi_0(x) = 2$. 6

(b) Solve the integral equation using Laplace transform

$$\varphi(x) = e^{2x} + \int_0^x e^{t-x}\varphi(t)dt.$$ 3

(c) Solve the integral equation

$$\varphi(x) - \lambda\int_{-\pi}^{\pi}(x\cos t + t^2\sin x + \cos x\sin t)\varphi(t)dt = 2x - \pi.$$ 5

UNIT–III

5. (a) Solve the integral equation of convolution type

$$\varphi(x) = xe^x - 2e^x\int_0^x e^{-t}\varphi(t)dt.$$ 6

(b) Using the Fredholm's determinant find the resolvent kernel of

$K(x,t) = \sin x\cos t, 0 \le x \le 2\pi, 0 \le t \le 2\pi$. 6

(c) Define integral equation with symmetric kernel, with example. 2

6. (a) Find the characteristic number and eigen function of the homogenous integral equation if the kernel is $K(x,t) = \begin{cases}(x+1)(t-2), 0 \le x \le t \\ (t+1)(x-2), t \le x \le 1\end{cases}$

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(b) Show that the integral equation

$\varphi(x)-\lambda\int_0^1\left(45x^2\ln t-9t^2\ln x\right)\varphi(t)dt=0$ does not have real characteristic number and eigen function. 6

UNIT–IV

7. (a) Prove that convolution is distributive with respect to addition. 4

(b) Find the value of $l^3\left\{n^2\sin nt\right\}$, where $l=$ integral operator. 4

(c) Prove that $\dfrac{\{t\}\left\{te^t\right\}}{\left\{e^t\right\}\{1\}}=\left\{e^t-1\right\}$. 4

(d) Show that $\left\{\dfrac{1}{\beta}e^{\alpha t}\sin\beta t\right\}=\dfrac{1}{(\mathbf{s}-\alpha)^2+\beta^2}$ where $\mathbf{s}=$ differential operator. 2

8. (a) Show that $\lim\limits_{\alpha-\infty}G_\alpha(x)=\lim\limits_{\alpha-\infty}\dfrac{\alpha}{\sqrt{\pi}}e^{-\alpha^2x^2}$ is a Dirac Delta function. 6

(b) Evaluate the following: 2×4=8

(i) $\delta\left(x^2-a^2\right)$

(ii) $\int_{-\infty}^{\infty}\delta(a-x)\delta(x-b)dx$

(iii) $\delta\left(x^2+x-2\right)$

(iv) $\int_{-2}^{\infty}e^t\delta(2t-4)\,dt$

UNIT–V

9. (a) Convert the differential equation $x^3y''+x^2y'-y=0$ to a Sturm-Liouville form. 3

(b) Find the eigen value and function of the Sturm-Liouville problem. 5

$$y'' + \lambda y = 0, 0 < x < L$$

$$y(0) = 0, hy(L) + y'(L) = 0, h > 0$$

(c) Express the function $f(x) = x$ as the eigen function series of the given Sturm-Liouville problem. 6

$$y'' + \lambda y = 0, 0 < x < 1$$

$$y(0) = 0, y(1) + y'(1) = 0$$

10. (a) Reduce the boundary value problem to an integral equation using Green's function 7

$$y'' + \lambda y = 2x + 1, 0 < x < 1$$

$$y(0) = y(1) = 0$$

(b) Using Green's function solve the boundary value problem 7

$$y'' - y = 0, 0 < x < 1$$

$$y(0) = y(1) = 0$$
