2024
M.Sc.
Second Semester
CORE – 07
MATHEMATICS
Course Code: MMAC 2.31
(Classical Mechanics)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Explain constraints of a dynamical system with examples. Formulate the equation of motion of a system of two masses, connected by an inextensible string passing over a smooth pulley. 3+5=8

   (b) Justify that the total angular momentum of a closed system is conserved due to isotropy of space. 6

2. (a) Distinguish between Newtonian, Lagrangian, and Hamiltonian mechanics. Prove that Lagrangian equations are invariant under Galilean transformation. 2+6=8

   (b) Define generalised potential. A particle moves in a plane under the influence of a force, acting towards a centre of force whose magnitude is $F = \frac{1}{r^2}\left(1 - \frac{\dot{r}^2 - 2r\ddot{r}}{c^2}\right)$, where r is the distance of the particle from the centre of force. Determine generalised potential that will result in from that force. Given that, c is the speed of the light and $\vec{F}$ is the force between two charges. 1+5=6

UNIT–II

3. (a) Define generalised momentum and cyclic co-ordinate. Justify that in absence of a given component of applied force the corresponding component of linear momentum is conserved. 1+1+6=8

(b) Derive the equation of motion of a compound pendulum by using Hamilton's equations. 6

4. (a) A dynamical system has the Lagrangian

$$L = \dot{q}_1^2 + \frac{\dot{q}_2^2}{a + bq_1^2} + k_1 q_1^2 + k_2 \dot{q}_1 \dot{q}_2 \text{ where } a, b, k_1 \text{ and } k_2 \text{ are constants.}$$

Obtain the equations of motion in Hamiltonian formalism. 8

(b) Explain Routhian. Find the Routhian for the Lagrangian L, given by

$$L = \frac{\mu}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{GMm}{r}, \text{ where } \mu = \frac{mM}{m+M}.$$ 2+4=6

UNIT–III

5. (a) Verify that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$, where H is the Hamiltonian function. Justify that Euler Lagrange equation can also be written in the form

$$\frac{d}{dx}\left(F - y'\frac{\partial F}{\partial y'}\right) - \frac{\partial F}{\partial x} = 0.$$ 2+5=7

(b) State modified Hamilton's principle. Deduce Hamilton's canonical equations from modified Hamilton's principle. 2+5=7

6. (a) Explain the method of Lagrange's undetermined multipliers in deriving the equation of motion for a conservative non-holonomic system from Hamilton's principle. 6

(b) What is Δ-variation? Discuss how it differs from δ-variation. Prove the principle of least action. 2+1+5=8

UNIT–IV

7. (a) What do you mean by canonical transformations? Formulate one-dimensional harmonic oscillator as an example of canonical transformations. 1+7=8

(b) Establish the relation between Lagrange bracket and Poison bracket. 6

8. (a) What do you mean by generating function? Verify that for the transformations $Q = \frac{1}{p}$ and $P = qp^2$ the bilinear form is invariant. 2+5=7

(b) State and discuss Liouville's theorem. 1+6=7

UNIT–V

9. (a) Define Euler's angles for the orientation of a rigid body and obtain an expression for the complete transformation matrix. 2+6=8

(b) Consider a rectangular parallelopiped of uniform density ρ mass M with sides a, b and c. For origin O at one corner, find the moment and product of inertia of the parallelopiped by taking the coordinate axes along the edges. If $a = b = c$ then determine the inertia tensor and principal inertia tensor. 3+3=6

10. (a) Formulate Euler's equation of motion for a rigid body by Lagrange's method. 7

(b) What do you mean by symmetric body? Discuss the theory of a force free spinning symmetrical top. 1+6=7