2024
M.Sc.
Second Semester
CORE – 05
MATHEMATICS
Course Code: MMAC 2.11
(Numerical Analysis)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Derive the secant method iteration formula. 4
   (b) Discuss the rate of convergence of Regula-Falsi method. 5
   (c) Find the cube root of 13 correct to four decimal places using Newton-Raphson method. 5

2. (a) Define absolute, relative, and percentage error. If $f(x, y, z) = 3\frac{xy}{z^3}$ and errors in x, y, z be 0.001. Compute the maximum relative error in $f(x, y, z)$ when $x = y = z = 1$. 3+2=5
   (b) Using Chebyshev's method, find the root of the equation $f(x) \equiv \cos x - xe^x = 0$ correct to six decimal places. (take $x_0 = 1.0$). 4
   (c) Find the root of the equation $f(x) \equiv x^4 - x - 10 = 0$ using multipoint iteration method. (perform four iteration) 5

UNIT–II

3. (a) Solve the system of equations by Cramer's rule: 5

$$x + 6y + 3z = 6$$
$$2x + 3y + 3z = 117$$
$$4x + y + 2z = 283$$

(b) Using Cholesky's method, find the inverse of the matrix: 9

$$\begin{bmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{bmatrix}$$

4. (a) Find the inverse of the matrix using partition method: 7

$$\begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(b) Find the largest and the smallest eigenvalue and its corresponding eigenvector of the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ using power method. 7

UNIT–III

5. (a) Using Newton's divided difference formula, find $y(10)$ given that $y(5) = 12, y(6) = 13, y(9) = 14, y(11) = 16.$ 4

(b) Use Bessel's formula to find $y(35)$, given that: 6

x	2.5	3.0	3.5	4.0	4.5	5.0
y	24.145	22.043	20.225	18.644	17.262	16.047

(c) Prove the following: 1×4=4

(i) If $f(x) = e^{ax}$, show that $\Delta^2 f(x) = (e^{ah} - 1)^n e^{ax}$

(ii) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

(iii) $\sqrt{1+\delta^2\mu^2} = 1 + \left(\frac{1}{2}\right)\delta^2$

(iv) $\delta = \Delta E^{-1/2} = \nabla E^{1/2}$

6. (a) Construct the Hermite interpolation that fits the data and interpolate at $x = 0.5$ and $x = 1.5$ 7

x	$f(x)$	$f'(x)$
0	4	-5
1	-6	-14
2	-22	-17

(b) Obtain the cubic spline interpolation for the data (taking $M_0 = M_4 = 0$)

x	1	2	3	4	5
$F(x)$	1	0	1	0	1

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UNIT–IV

7. (a) Given the distance x cm for various values of time t seconds of a particle below. Find the velocity and acceleration when t=0.3 second

Time (sec)	0	0.1	0.2	0.3	0.4	0.5	0.6
Velocity (m/Sec)	30.1	31.6	32.9	33.6	34	33.8	33.2

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(b) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ of $y = x^{1/3}$ at $x = 50$, from the table below: 7

x	50	51	52	53	54	55	56
y	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

Compute the magnitudes of errors.

8. (a) Compute the value of the definite integral $\int_{0.2}^{1.4} \frac{dx}{4x+5}$ by trapezoidal rule and Simpson's 1/3 rule and hence calculate the absolute error in each case, (take $h = 0.2$) 6

(b) Derive the Gauss two point rule, hence find the value of the integral

$I = \int_0^2 \frac{dx}{3+4x}$ using Gauss two point rule. Compare with the exact solution. 8

UNIT–V

9. (a) Using Euler's method, find the solution of the equation

$\frac{dy}{dx} = x + \sqrt{y}, y(0) = 1$ for the range $0 \le x \le 0.6$ with $h = 0.2$. 4

(b) Consider the initial value problem $\frac{dy}{dx} = x(y+1), y(0) = 1$ compute $y(0.5)$ with $h = 0.1$ using mid point method. If the exact solution is $y = -1 + 2e^{x^2/2}$. Find the magnitude of actual errors. 5

(c) Solve the initial value problem $\frac{dy}{dx} = 2x + 3y, y(0) = 1$, using Taylor series method with $h = 0.2$ over the interval $[0,1]$. 5

10. (a) For the initial value problem $y' = x - y^2, y(0) = 0$ apply Milne-Simpson method in the interval $0 \le x \le 1$. (Assume $h = 0.25$). 6

(b) Reduce the initial value problem $y''' + y'' = x(y')^2 - y^2$, with $y(0) = 1, y'(0) = 0$ to a system of first order initial value problems and find the value of $y(1), y'(1) (h = 0.5)$ using Runge-Kutta method of fourth order. 8