2024
B.A./B.Sc.
Second Semester
GENERIC ELECTIVE – 2
STATISTICS
Course Code: STG 2.11
(Introductory Probability)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Explain in detail the scatter diagram. 4
   (b) Obtain the normal equations for estimating a and b from the line of regression $Y = a + bx$. 5
   (c) Show that correlation coefficient lies between –1 and +1. 5

2. (a) Write the assumptions of Karl Pearson's correlation coefficient. If $Z = aX + bY$ and r is the correlation coefficient between X and Y, prove that: $\sigma_Z^2 = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2abr\sigma_X\sigma_Y$. 2+4=6
   (b) Write the properties of regression coefficient and prove any two properties of regression coefficient. 2+3+3=8

UNIT–II

3. (a) Derive the correlation coefficients in terms of the total correlation coefficient between the pairs of variables. 6
   (b) Give the equations for estimate and residual as given by the plan of regression. 2
   (c) From the data relating to the yield of dry bark (X_1), height (X_2) and girth (X_3) for 18 cinchona plants, the following correlation coefficients were obtained: $r_{12} = 0.77$, $r_{13} = 0.72$ and $r_{23} = 0.52$.
   Find the partial coefficient $r_{12.3}$ and multiple correlation coefficients $R_{1.23}$. 6

4. (a) Prove that the partial correlation coefficient between X_1 and X_2 is

$$r_{12.3}=\frac{r_{12}-r_{13}r_{23}}{\sqrt{\left(1-r_{13}^2\right)\left(1-r_{23}^2\right)}}.$$ 6

(b) Show that: $1-R_{1.23}^2=\left(1-r_{12}^2\right)\left(1-r_{13.2}^2\right)$, deduce that: 3+2+3=8

(i) $R_{1.23}^2=r_{12}^2+r_{13}^2$, if $r_{23}=0$

(ii) $1-R_{1.23}^2=\frac{(1-\rho)(1+2\rho)}{(1+\rho)}$, provided all coefficient of zero order are equal to ρ.

UNIT–III

5. (a) State and prove the addition theorem of probability. 4

(b) Define continuous distribution function. Write any of its four properties. 4

(c) A variable X is distributed at random between the values 0 and 4 and its probability density function is given by $f(x)=kx^3(4-x)^2$. Find the value of k, the mean and standard deviation of the distribution. 6

6. (a) What is the expectation of the number of failures preceding the first success in an infinite series of independent trials with constant probability p of success in each trials? 4

(b) State and prove the weak law of large numbers. 5

(c) For the joint probability distribution of two random variables X and Y given below: 2½×2=5

$Y\downarrow$ \ $X\rightarrow$	1	2	3
1	$\frac{5}{27}$	$\frac{4}{27}$	$\frac{2}{27}$
2	$\frac{1}{27}$	$\frac{3}{27}$	$\frac{3}{27}$
3	$\frac{3}{27}$	$\frac{4}{27}$	$\frac{2}{27}$

Find:

(i) Marginal distribution of X and Y

(ii) The conditional probability distribution X given $Y = 1$

UNIT–IV

7. (a) Define binomial distribution. Derive the theorem for the additive property of binomial distribution. 5

(b) If 5 cards are drawn from a pack of 52 playing cards, what is the probability that 3 will be hearts? 3

(c) Write any two physical condition of binomial distribution. 2

(d) A discrete random variable which follows a hypergeometric distribution with probability mass function $p(X = k) = h(k; N, M, n)$. Find the variance of this distribution. 4

8. (a) Define Poisson distribution. State and prove the additive property of Poisson distribution. 1+4=5

(b) Deduce the formula for skewness and kurtosis of Poisson distribution. 1½+1½=3

(c) Find the first and second moments of geometric distribution. 3+3=6

UNIT–V

9. (a) Define normal distribution. For a normal distribution, prove that it's mean is equal to median i.e. mean = median = μ. 1+6=7

(b) Let $X \sim U(a,b)$. Then its probability density function is given by

$f(x) = \frac{1}{b-a}; a < x < b$. Find the first two moment's i.e. μ'_1 and μ_2. 3½+3½=7

10. (a) Write the probability density function of exponential distribution. For an exponential distribution, prove that $E(X) = \frac{1}{\lambda}$ and $Var(X) = \frac{1}{\lambda^2}$. 1+6=7

(b) Derive the theorem for mean deviation about mean of uniform distribution. 3

(c) For a normal distribution, if $X \sim N(\mu, \sigma^2)$. Then prove that the standard normal variate Z is given by $Z \sim N(0,1)$. 4
