2024
B.A./B.Sc.
Fourth Semester
CORE – 9
STATISTICS
Course Code: STC 4.21
(Linear Models)

Total Mark: 70   *Pass Mark: 28*
Time: 3 hours

Answer <u>five</u> *questions, taking* <u>one</u> *from each unit.*

UNIT–I

1. (a) For an estimable function $\tilde{l}^T\tilde{\beta}$, prove that $Rank(A) = Rank\begin{pmatrix} A \\ \tilde{l}^T \end{pmatrix}$.
                                                                                                        5

    (b) Define the following: 2×2=4
        (i) Parametric function
        (ii) Estimable function

    (c) Prove the theorem which is fundamental to analysis of variance called linear model.   5

2. (a) Define minimum variance unbiased estimator in linear estimation. How do we choose the variance in linear estimation? Explain them.
                                                                                                2+3=5

    (b) Let $Y_1, Y_2,..., Y_n$ be a set of n independent random variables with a common variance σ^2 and coefficients of $a'_{ij} s$ of p unknown parameters $\beta_1, \beta_2,...,\beta_p$, then prove that

    $D(\tilde{Y}) = E[Y - E(Y)][Y - E(Y)] = \sigma^2 I$, where each of them has its usual meaning.   6

    (c) For a Markov linear model, define fixed effect model or model-I.   3

UNIT–II

3. (a) Describe the natures of the indicator variables in case of analysis of variance and regression analysis. Describe the test for the relationship between two variables using analysis of variance technique.   2+5=7
   (b) Using the technique of analysis of variance, test for the equality of regression equations from p groups.   7

4. (a) What are the types of values taken by the indicator variables in case of analysis of covariance? Use the analysis of variance technique to test if the regression line is linear.   1+6=7
   (b) Delineate the method of analysis of variance in testing the hypothesis that there is no dependence of the dependent variable y on the dependent variable x in case of a multiple linear regression model.   7

UNIT–III

5. (a) Define the linear models in respect of analysis of variance (ANOVA) technique. Also define all the linear models in ANOVA.   4+4=8
   (b) For a fixed additive model, $y_{ij} = \mu + \alpha_i + \epsilon_{ij}$, estimate $\hat{\mu}$, $\hat{\alpha}$ and $\hat{\epsilon}_{ij}$. Also prove that total $SS = S_C^2 + S_E^2$, where each term has its usual meaning.   3+3=6

6. (a) From an ANOVA define the relation for mean sum of squares. Also, explain how to conclude for calculated F and tabulated F.   5
   (b) Explain in detail the analysis of variance (ANOVA) technique.   3
   (c) Give the layout for analysis of variance (ANOVA). Also, define the values of G_i, G, $\bar{y}_{i0}$ and $\bar{y}_{00}$.   6

UNIT–IV

7. (a) For analysis of covariance technique, let Y be obtained from linear regression of Y on X, which is given by $\hat{Y} = \bar{Y} + b_{yx}(X - \bar{X})$, then prove that $Y = SS_{YY} - b_{yx}SS_{XY}$.   6

(b) Let $Q = \sum_{i=1}^{k} \sum_{j=1}^{n_i} e_{ij}^{*2}$, prove that $\hat{\mu}_i = \bar{y}_{i0} - \hat{\beta}(\bar{x}_{i0} - \bar{x}_{00})$ and

$$\hat{\beta} = \frac{E_{XY}}{E_{XX}}.$$

5

(c) Give the formula for partition of total variation in Y and X. 3

8. (a) Derive the relation for unrestricted residual sum of squares for model-I. Also, give the null hypothesis and alternate hypothesis for analysis of variance. 6

(b) Give the statistical analysis for analysis of covariance for randomized block design. 8

UNIT–V

9. (a) What is theoretical econometrics? Explain the various steps involved in the methodology of econometrics. 2+3=5

(b) What do you mean by multicollinearity? 2

(c) Explain the Durbin-Watson and Run's test for detecting autocorrelation. 7

10. (a) Explain the specification of the econometric model of consumption. 3

(b) Prove that if the intercorrelation between the explanatory variables is perfect $\left(r_{x_i x_j} = 1 \right)$, then the estimates of the coefficients are indeterminate. 4

(c) Define marginal propensity to consume with example. 3

(d) Prove the mean, variance, and covariance of autocorrelation. 4

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