2024

B.A./B.Sc.

Fourth Semester

CORE – 8

STATISTICS

Course Code: STC 4.11

(Statistical Inference)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) What do you understand by parameter space? In particular, discuss the requirements of consistency and unbiasedness of an estimate. 1+3=4

(b) Let X be distributed in the Poisson form with parameter Θ. Show that only unbiased estimator of $\exp\{-(k+1)\theta\}, k>0$, is $T(X) = (-K)^X$ so that $T(x) > 0$ if x is even and $T(x) < 0$ if x is odd. 3

(c) If T_1 and T_2 are two unbiased estimators of $\gamma(\theta)$, having the same variance and ρ is the correlation between them, then show that $\rho \geq 2e-1$, where e is the efficiency of each estimator. 4

(d) Let $X_1, X_2, ..., X_n$ be a random sample from a population with p.d.f: $f(x,\theta) = \theta x^{\theta-1} : 0 < x < 1, \theta > 0$. 3

2. (a) Define the minimum variance unbiased estimator. 2

(b) Prove the Invariance property of a consistent estimators. 4

(c) A random sample $(X_1, X_2, X_3, X_4, X_5)$ of size 5 is drawn from a normal population with unknown mean μ. Consider the following estimators to estimate μ:

(i) $t_1 = \dfrac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$

(ii) $t_2 = \frac{X_1 + X_2}{2} + X_3$

(iii) $t_3 = \frac{2X_1 + X_2 + \lambda X_3}{3}$

Where λ is such that t_3 is an unbiased estimator of μ. Find λ. Are t_1 and t_2 unbiased? State giving reasons, the estimator which is best among t_1, t_2 and t_3. 5

(b) Let $x_1, x_2, ..., x_n$ be a random sample from a uniform population on $[0, \theta]$. Find a sufficient estimator for θ. 3

UNIT–II

3. (a) Let $X_1, X_2, ..., X_n$ be a set of random variable taken from a population described by the joint probability function $f_\theta(x_1, x_2, ..., x_n)$ where $\theta \in \Theta$. Let $T = t(X_1, X_2, ..., X_{n)}$ be an unbiased estimator of θ, say $\tau(\theta)$. Then prove that,

$$Var(t) \geq \frac{\{\tau'(\theta)\}^2}{E_\theta\left\{\frac{\delta^2}{\delta\theta^2} \log f_\theta(x_1, x_2, ..., x_n)\right\}}$$ 7

(b) For an interval estimation, construct a general term for confidence limit and confidence interval at 99% level. For a large sample from a normal distribution with mean μ and standard deviation σ and

$$Z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} \sim N(0,1).$$ 7

4. (a) A random sample $(X_1, X_2, ..., X_n)$ is taken from a normal distribution $N(0, \sigma^2)$. Examine if $\sum_{i=1}^{n} \frac{x_i^2}{n}$ is an MVB estimator for σ^2. 5

(b) Prove that $E[\text{Ø}(X)] = E(Y) = \mu$. 2

(c) A random sample of 85 observations has mean 20 and a standard deviation 3.5. Construct 99% confidence interval for the population mean. 7

UNIT–III

5. (a) In a random sampling from normal population $N(\mu, \sigma^2)$, find the M.L.E for 3×2=6
   (i) μ when σ^2 is known
   (ii) σ^2 when μ is known

   (b) Explain the method of maximum chi-square (χ^2). 4

   (c) Present the general structure of Baye's estimators and give its two properties. 2+2=4

6. (a) Define likelihood function. What properties of estimators are being usually held by maximum likelihood estimators? 1+3=4

   (b) For two Poisson distribution:

$$p(x) = P(X = x) = \frac{1}{2} \cdot \frac{e^{-m_1} . m_1^x}{x!} + \frac{1}{2} \cdot \frac{e^{-m_2} . m_2^x}{x!} \forall x = 0, 1, 2, ...$$

   Show that the estimate for m_1 and m_2 by the method of moments are:

   $\mu_1' \pm \sqrt{\mu_2' + \mu_1' + \mu_1'^2}$. 6

   (c) Write the properties of the estimates obtained by the method of minimum chi-square. Compare the method of minimum chi-square with maximum likelihood estimation. 2+2=4

UNIT–IV

7. (a) Write a note on the following: 1+1+2=4
   (i) Critical region
   (ii) Power of the test
   (iii) Two types of errors

   (b) If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from the population: $f(x, \theta) = \theta \exp(-\theta x); 0 \leq x \leq \infty$.
   Obtain the value of type I and type II errors. 5

   (c) Explain the test for the mean of a normal population. 5

8. (a) Define uniformly most powerful test (UMP). 2

(b) State Neyman Pearson Lemma. Obtain the region for testing $\theta = \theta_0$ against $\theta = \theta_1 > \theta_0$ and $\theta = \theta_1 < \theta_0$, in the case of a normal population with mean θ and variance σ^2 where σ^2 is known using Neyman Pearson Lemma. 2+6=8

(c) Define parameter space. Examine whether a best critical region exists for testing the null hypothesis $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta > \theta_0$ for the parameter θ of the distribution: 1+3=4

$$f(x,\theta) = \frac{1+\theta}{(x+\theta)^2}, 1 \le x \le \infty .$$

UNIT–V

9. (a) Write the advantages and disadvantages of non-parametric method. 2+2=4

(b) Stating the underlying assumptions and the null hypothesis, develop Kruskal Wallis test. 5

(c) Explain test for randomness and empirical distribution function. 2+3=5

10. (a) Distinguish between parametric and non-parametric tests. 4

(b) How can one use Mann-Whitney test for two sample problem? 5

(c) Explain sign test for one sample and give some of its uses. 5