2024
B.A./B.Sc.
Second Semester
CORE – 4
STATISTICS
Course Code: STC 2.21
(Algebra)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Solve the equation $2x^3 + x^2 - 7x - 6 = 0$, when the difference of two roots is 3. 4

(b) If $\alpha, \beta, \gamma, \delta$ be the roots of the biquadratic equation $x^4 - px^3 + qx^2 - rx + s = 0$, then find in terms of p, q, r, s the values of the following: 2×3=6

(i) $\sum \alpha^2\beta$ (ii) $\sum \alpha^2\beta\gamma$ (iii) $\sum \frac{1}{\alpha^2}$

(c) Solve the equation $x^5 - x^4 + 8x^2 - 9x - 15 = 0$, it is given that the root being $\sqrt{3}$ and (1–2). 4

2. (a) Transform the equation whose roots are those of $3x^3 - 2x^2 + x - 9 = 0$, each diminish by 5. 2

(b) Solve the equation $x^4 + 15x^3 + 70x^2 + 120x + 64 = 0$, whose roots are in G.P. 4

(c) Solve the equation $x^3 - 7x - 6 = 0$, given that the roots are connected by the relation $\alpha + 2\beta = 1$. 4

(d) Use synthetic division to find the quotient in each problem: 2×2=4

(i) $\dfrac{x^4 - 15x^2 + 10x + 24}{x + 4}$ (ii) $\dfrac{3x^4 - 8x^3 + 9x^2 - 2x - 2}{3x + 1}$

UNIT–II

3. (a) Define conjugate and conjugate transpose of a matrix with examples. 3

(b) Prove that the product of the matrices

$$A=\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix},\ B=\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

is zero when θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$. 4

(c) Define idempotent and nilpotent matrices with examples. 3

(d) If $A=\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, show that $A^k=\begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$. 4

4. (a) Define Hermitian and Skew-Hermitian matrices. 3

(b) Show that for any three matrices *A*, *B* and *C*, A(B+C)=$AB+AC$. 3

(c) Define the adjoint of a square matrix with an example. 4

(d) Find the inverse of the matrix $A=\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$, using Adj. *A*. 4

UNIT–III

5. (a) If any two rows (or two columns) of a determinant are interchanged, the value of the determinant is multiplied by –1. 3

(b) Show that $\Delta=\begin{vmatrix} a_1+\alpha_1 & b_1 & c_1 \\ a_2+\alpha_2 & b_2 & c_2 \\ a_3+\alpha_3 & b_3 & c_3 \end{vmatrix}=\begin{vmatrix} a_1 & b_1 & c_1 \\ b_1 & b_2 & c_2 \\ c_1 & b_3 & c_3 \end{vmatrix}+\begin{vmatrix} \alpha_1 & b_1 & c_1 \\ \alpha_2 & b_2 & c_2 \\ \alpha_3 & b_3 & c_3 \end{vmatrix}$. 3

(c) Prove that $\Delta=\begin{vmatrix} 1 & a & a^2 & a^3+bcd \\ 1 & b & b^2 & b^3+cda \\ 1 & c & c^2 & c^3+dab \\ 1 & d & d^2 & d^3+abc \end{vmatrix}=0$. 4

(d) Solve by Cramer's rule:

$$x+2y+3z=6$$
$$2x+4y+z=7$$
$$2x+2y+9z=14$$

4

6. (a) Show that the value of the determinant of a skew symmetric matrix of odd order is always zero. 3

(b) Prove that $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$. 4

(c) Solve the equation $\begin{vmatrix} 2x-5 & 2 & 2 \\ 2 & 2x-5 & 2 \\ 2 & 2 & 2x-5 \end{vmatrix} = 0$. 3

(d) Prove that

$$\Delta = \begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right).$$ 4

UNIT–IV

7. (a) Define rank of a matrix. Reduce the matrix A to its normal form and find the rank, where $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$. 2+4=6

(b) Prove that $\text{Rank}(AA^{\Theta}) = \text{Rank}(A)$. 3

(c) Compute the inverse of the matrix $A = \begin{bmatrix} -1 & -3 & -3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 1 & 0 \end{bmatrix}$, using the method $I = BA$, where B is the inverse of A. 5

8. (a) Show that the rank of any matrix rank $\geq$ of its every sub-matrix. 2

(b) If A is an n-square matrix of rank$(n–1)$, show that $\text{Adj}.A \neq 0$. 2

(c) Reduce the matrix $A=\begin{bmatrix}1 & -1 & 3 & 6\\ 1 & 3 & -3 & 4\\ 5 & 3 & 3 & 1\end{bmatrix}$ to its normal form and hence find its rank. 3

(d) Show that the rank of a matrix is equal to the rank of the transposed matrix. 2

(e) Solve, with the help of the matrices, the simultaneous equations: 5

$x+y+z=3$

$x+2y+3z=4$

$x+4y+9z=6$

UNIT–V

9. (a) Define subspace. Let U be the set of all vectors of the form

$\begin{bmatrix}2r-s\\ 2\\ r+s\end{bmatrix}$, $r, s \in \mathbb{R}$. Is U a replacement of $\mathbb{R}^3$? 1+3=4

(b) Determine whether the vectors $v_1=(3, 1, -4)$, $v_2=(2, 2, -3)$, $v_3=(0, -4, 1)$ are linearly independent. If so, find the relation between them. 4

(c) State Caley-Hamilton theorem. Verify the Caley-Hamilton theorem for the matrix $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$, hence find A^{-1}. 6

10. (a) Define linear dependence and independence of vectors with examples. 4

(b) Prove that a set of non-zero vectors $v_1, v_2, ..., v_n \in V$ is linearly dependent if and only if one of them is a linear combination of the other or preceding vector. 4

(c) Define the characteristic roots and determine the characteristic roots of the matrix $A=\begin{bmatrix}-3 & -9 & -12\\ 1 & 3 & 4\\ 0 & 0 & 1\end{bmatrix}$. 2+4=6