2024
B.A./B.Sc.
Second Semester
CORE – 3
STATISTICS
Course Code: STC 2.11
(Probability Distributions & Correlation Analysis)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) State and prove multiplication theorem of probability. 4
   (b) A coin is tossed until a head appears. What is the expectation of the number of tosses required? 4
   (c) Define moment generating function of a random variable of *X*. If *M* (*t*) is the m.g.f. of a random variable of *X* about the origin. Show that the moment μ'_r is given by $\mu'_r = \left[\frac{d'\mu(t)}{dt^r}\right]_{t=0}$. 6

2. (a) State and prove the Chebychev's inequality. 6
   (b) If *X* and *Y* are two independent random variables, show that
   $Var(aX + bY) = a^2Var(X) + b^2Var(Y)$ 4
   (c) Consider the following probability distribution

$X\downarrow / Y\rightarrow$	0	1	2
0	0.1	0.2	0.1
1	0.2	0.3	0.1

   (i) Show that *X* and *Y* have different expectations. 2
   (ii) Find *Var*(*X*) and *Var*(*Y*). 2

UNIT–II

3. (a) Obtain Poisson distribution as a limiting case of binomial distribution. 5

(b) Explore the additive property of binomial distribution and make a comment on it. For a binomial distribution the mean is 6 and the standard deviation is $\sqrt{2}$. Write out all the terms of the distribution. 3+2=5

(c) If X is a Poisson variate such that
$P(X=2)=9P(X=4)+90P(X=6)$. Find

(i) λ, the mean of X 2

(ii) β, the coefficient of skewness 2

4. (a) Define hyper-geometric distribution. Obtain the mean and variance of a hypergeometric distribution. 2+3=5

(b) Obtain the recurrence relation for the probabilities of binomial distribution. Also, comment on its use for fitting of binomial distribution. 3+3=6

(c) A student takes a true-false examination consisting of 10 questions. He is completely unprepared so he plans to guess each answer. The guesses are to be made at random. For example, he may toss a fair coin and use the outcome to determine his guess.

(i) Compute the probability that he guesses correctly at least five times. 1

(ii) Compute the probability that he guesses correctly at least 9 times. 1

(iii) What is the smallest n that the probability of guessing at least n correct answers is less than ½. 1

UNIT–III

5. (a) Define uniform distribution. Also, give its cumulative distribution function. 4

(b) If X_1 and X_2 are two independent rectangular variates on (0, 1), find the distribution of: 2+2=4

(i) $\dfrac{X_1}{X_2}$ (ii) X_1X_2

(c) Discuss the important properties of normal distribution. 6

6. (a) Define exponential distribution. Obtain its moment generating function. 2+3=5

(b) Obtain the median and mode of normal distribution and comment on it. 2+2=4

(c) Define gamma distribution. Obtain the m.g.f. and cumulant generating function of gamma distribution. 1+2+2=5

UNIT–IV

7. (a) Define correlation. X and Y are two random variables with variances σ_X^2 and σ_Y^2 respectively and 'r' is the coefficient of correlation between them. If $U = X + KY$ and $V = X + \left(\frac{\sigma_X}{\sigma_Y}\right)Y$, find the value of K so that U and V are uncorrelated. 1+5=6

(b) Write the properties of regression coefficients. Prove one of the properties of regression coefficients. 2+2=4

(c) Fit a second degree equation $Y = a + bX + cX^2$ by the method of least square. 4

8. (a) Define rank correlation coefficient. Prove Spearman's formula for the rank correlation coefficient. 1+4=5

(b) Prove that regression coefficients are independent of the change of origin but not of scale. 5

(c) Define the line of regression. How is the line of regression is obtained? Give the normal equations of $Y = aX^b$. 1+1+2=4

UNIT–V

9. (a) Show that the variance of the residual for a tri-variate distribution is

$$\sigma_{1.23}^2 = \sigma_1^2 \frac{w}{w_{11}}.$$ 6

(b) If $1-R_{1.23}^2=(1-r_{12}^2)(1-r_{13.2}^2)$, then prove that

$$1-R_{1.23}^2=\frac{(1-\rho)(1+2\rho)}{(1+\rho)}$$, provided all coefficients of zero order are equal to ρ. 3

(c) What do you mean by independence of attributes? Give a criterion of independence for attributes *A* and *B*. And also find if *A* and *B* are independent, positively associated or negatively associated in the case: *N*=1000, (*a*)= 470, (*b*)= 620 and (*AB*)= 320. 1+2+2=5

10. (a) Show that the correlation coefficients between the residuals $X_{1.23}$ and $X_{2.13}$ is equal and opposite of that between $X_{1.3}$ and $X_{2.3}$. 5

(b) Define coefficient of partial correlation and prove that partial correlation coefficient between X_1 and X_2 is

$$r_{12.3}=\frac{Cov(X_{1.3},X_{2.3})}{\sqrt{Var(X_{1.3})Var(X_{2.3})}}$$ 1+5=6

(c) Define the following: 1×3=3

(i) Dichotomy

(ii) Class frequencies

(iii) Consistency of data