2024

B.A./B.Sc.

Sixth Semester

DISCIPLINE SPECIFIC ELECTIVE – 3

PHYSICS

Course Code: PHD 6.11

(Advanced Mathematical Physics – II)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Create the equation of motion of a system defined by time dependent Lagrangian $L = e^{bt}\left[\frac{1}{2}m\dot{x}^2 - v(x)\right]$, here '*b*' is a constant. 3

   (b) Use Lagrange's equations to find the equation of motion of a compound pendulum in a vertical plane about a fixed horizontal axis. Hence find the angular frequency of small amplitude oscillations of the compound pendulum. 5

   (c) Arrive at Euler-Lagrange equation using calculus of variation of a curve. 6

2. (a) If $I(y) = \int_0^1 (x\sin y + \cos y)dx$ and $y(0) = 0, y(1) = \frac{\pi}{4}$. Find the extremal. 4

   (b) Write a short note on virtual work and explain D'Alembert's principle. 5

   (c) Two particles of masses m_1 and m_2 move under the action of their gravitational interaction, find the Lagrangian equations of motion of the particles. 5

UNIT–II

3. (a) Use Hamilton's principle to find the equation of motion of one-dimensional harmonic oscillator. 4

(b) Show that transformation defined by
$q = \sqrt{2P}\sin Q, P = \sqrt{2P}\cos Q$ is canonical by using Poisson bracket. 4

(c) Obtain the Lagrangian, Hamiltonian and equations of motion for a projectile near the face of the earth. 6

4. (a) Using Hamilton's equations of motion, show that the Hamiltonian

$$H = \frac{p^2}{2m}e^{-rt} + \frac{1}{2}m\omega^2 x^2 e^{rt}$$ leads to the equation of motion of a damped harmonic oscillator $\ddot{x} + r\dot{x} + \omega^2 x = 0$. 4

(b) If r, θ are the polar coordinates of a particle of mass m, find the Hamilton's equations. 4

(c) Show that Lagrange bracket is invariant under canonical transformation. 6

UNIT–III

5. (a) If $G = \{ f_1, f_2, f_3, f_4 \}$ four functions defined by

$$f_1(x) = x,\ f_2(x) = -x,\ f_3(x) = \frac{1}{x},\ f_4(x) = -\frac{1}{x}; \forall \in R - \{0\}$$. Show that G is an abelian group. 6

(b) Find all the generators of the cyclic group
$\{a, a^2, a^3, a^4, a^5, a^6, a^7, a^8 = e\}$ of order 8. 8

6. (a) Show that the generator of a cyclic group of order 'n' are the generator a^P and $0 < P < n$. 6

(b) Differentiate between homomorphism and isomorphism of group. Find the permutation group isomorphic to the group
$(\{1, -1, i.-i\}, x)$. 3+5=8

UNIT–IV

7. (a) A die is tossed. If the number is odd, what is the probability that it is prime? 4

(b) Prove Baye's theorem. 4

(c) Determine the coefficient of $a^2 b^3 c^2 d^5$ in the expansion of
$(a + 2b - 3c + 2d + 5)^{16}$. 6

8. (a) If the discrete random variable has

X	1	2	3	4	5	6	7
$P(X=x)$	k	$2k$	$3k$	k^2	k^2+k	$2k^2$	$4k^2$

Find mean and variance of the above probability distribution. 4

(b) Out of 800 families with 4 children each, how many families would be expected to have; 4

(i) At least 1 boy

(ii) 2 boys and 2 girls

(c) Find the probability distribution of number of doublets in three throws of a pair of a die. 6

UNIT–V

9. (a) Calculate the first four moments about 6 for 7, 9, 8, 6. 4

(b) If the height of 300 students are normally distributed with mean 64.5 inches and standard deviation 3.3 inches. How many students have height 4

(i) less than 5 feet

(ii) between 5 feet and 5 feet 9 inches

(c) Find the mode of binomial distribution. 6

10. (a) Prove that 1st moment of binomial distribution is $\mu'_1 = np$. Here, n : number of trail and p : number of success. 4

(b) Using the method of least square, fit a straight line to the four points.

x	1	2	3	4
y	1.7	1.8	2.3	3.2

4

(c) In a certain factory turning out razor blades, there is a chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate no. of packets containing no defective, one defective and 2 defective blades respectively in a consignment of 10,000 packets. 6