2024
B.A/B.Sc.
Fourth Semester
CORE – 8
PHYSICS
Course Code: PHC 4.11
(Mathematical Physics - III)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Find the value of $(1+i)^{1/5}$. 4

   (b) Solve the equation with the help of De Moivre's theorem $x^7 - 1 = 0$. 5

   (c) If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\phi = y + \frac{1}{y}$, then prove that $x^p.y^q + \frac{1}{x^p.y^q} = 2\cos(p\theta + q\phi)$. 5

2. (a) Show that the real and imaginary parts of the function $w = \log z$ satisfy the Cauchy-Riemann equations when z is not zero. Find its derivative. 6

   (b) Which of the following function is analytic? 8

   (i) $x^2 + iy^2$ (ii) $2xy + i(x^2 - y^2)$

UNIT–II

3. (a) Evaluate $\int_0^{1+i} (x^2 - iy)\,dz$, along the path

   (i) $y = x$ (ii) $y = x^2$ 6

   (b) Expand the following function in Taylor series e^z about a. 4

(c) Using Cauchy integral formula to calculate $\int_C \frac{2z+1}{z^2+z}dz$, where C is circle $|z|=\frac{1}{2}$. 4

4. (a) Determine the poles and residue of the function $f(z)=\cot z$. 4

(b) Using Residue theorem, evaluate $\oint_c \frac{z^2+1}{z^2-1}dz$, where C is the circle $|z|=\frac{3}{2}$. 4

(c) Evaluate $\int_0^{2\pi} \frac{d\theta}{a+b\sin\theta}$ if $a>|b|$. 6

UNIT–III

5. (a) Find the Fourier sine integral for $f(x)=e^{-\beta x}\,(\beta>0)$. Hence show that $\frac{\pi}{2}e^{-\beta x}=\int_0^{\alpha}\frac{\lambda\sin\lambda x}{\beta^2+x^2}d\lambda$ 5

(b) Find the Fourier transform of $f(x)=\begin{cases}1-x^2, & \text{if } |x|\le 1\\ 0, & \text{if } |x|>1\end{cases}$ 5

(c) Find the Fourier transform of e^{-ax^2}, where $a>0$. 4

6. (a) Obtain Fourier cosine transform of $f(x)=5e^{-2x}+2e^{-5x}$. 6

(b) Show that $F\{f(x-a)\}=e^{isa}F(s)$, if $F(x)$ is complex Fourier transform of $f(x)$. 3

(c) Show that $\int_{-\alpha}^{\alpha}\delta(x).dx=\begin{cases}\frac{1}{2} & \text{if } a>0\\ -\frac{1}{2} & \text{if } a<0\end{cases}$ 5

UNIT–IV

7. Find the Laplace transforms of

(a) $f(t)=\begin{cases}\dfrac{t}{k}, & \text{when } 0<t<k\\ 1, & \text{when } t>k\end{cases}$ 4

(b) $f(t)=4\cosh 2.\sin 4t$ 4

(c) $f(t)=\sin\sqrt{t}$, hence find $L\left(\dfrac{\cos\sqrt{t}}{2\sqrt{t}}\right)$ 6

8. (a) Find the Laplace transform of the periodic function

$$f(t)=\begin{cases}t, & 0<t<\pi\\ \pi-t, & \pi<t<2\pi\end{cases}$$ 6

(b) Express the following function in terms of unit step functions and find its Laplace transform: $f(t)=\begin{cases}8, & t<2\\ 6, & t\geq 2\end{cases}$ 4

(c) Find the inverse Laplace transform of $\dfrac{s}{4s^2-25}$. 4

UNIT–V

9. (a) Solve $\dfrac{\partial u}{\partial t}=k\dfrac{\partial^2 u}{\partial x^2}$ for $0\leq x\leq \alpha, t>0$ under the given conditions: 7

(i) $u(x,0)=0$ for $x\geq 0$

(ii) $\dfrac{du(0,t)}{du}=-a$ (constant)

(b) Voltage Ee^{-at} is applies at $t = 0$ to a. Circuit of inductance L and resistance R. Show that the current at time t is $\frac{E}{R-aL}\left(e^{-at} - e^{-\frac{Rt}{L}}\right)$. 7

10. (a) Using the Laplace transform, find the current $i(t)$ in the LC-circuit. Assuming $L = 1$ henry, $C = 1$ farad, zero initial current and charge on the capacitor, and $V(t) = \begin{cases} t, & \text{when } 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$ 6

(b) Using Laplace transform, find the solution of the initial value problem of: 4×2=8

(i) $y'' - 4y' + 4y = 64 \sin 2t$, such that, $y(0) = 0, y'(0) = 1$

(ii) $y'' + 4y' + 4y = 6e^{-t}$, such that, $y(0) = -2, y'(0) = 8$