2024

B.A./B.Sc.

Fourth Semester

SKILL ENHANCEMENT COURSE – 2

MATHEMATICS

Course Code: MAS 4.11

(Graph Theory)

Total Mark: 35 *Pass Mark: 14*

Time: 2 hours

Answer five *questions, taking* one *from each unit.*

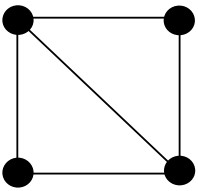
UNIT–I

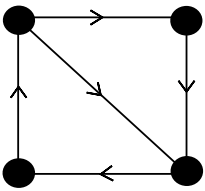
1. (a) Define a simple graph. What is the maximum number of edges in a simple graph with n vertices? Justify your answer. 1+3=4
   (b) Show that the total number of odd vertices of a graph is always even. 3

2. (a) State Hakimi-Havel theorem. Show that the sequence [6, 5, 5, 4, 3, 3, 2, 2, 2] is a graphical vector. Also draw its corresponding graph. 1+2+1=4
   (b) Find the number of subgraphs and spanning subgraphs in K_6. 2
   (c) Define source and sink. 1

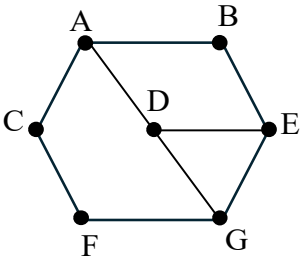
UNIT–II

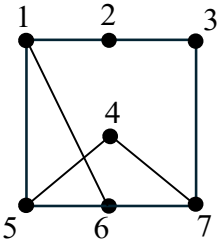
3. (a) Define compliment of a simple graph. Draw two self compliment graph of order 5. 1+2=3
   (b) Construct the adjacency and incidence matrix to represent the graph: 2+2=4

G_1: 

G_2: 

4. (a) Show that the graph are isomorphic using adjacency matrix: 4



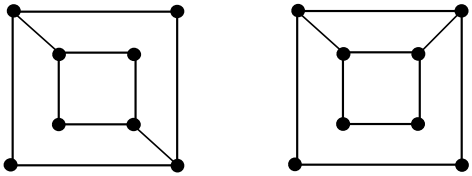


(b) Construct the pseudograph represented by the incidence matrix: 3

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

UNIT–III

5. (a) Show that the following graphs are not isomorphic: 3

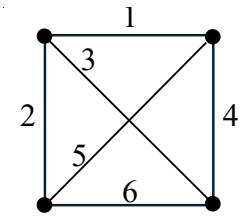


(b) Prove that the maximum edges of a simple graph with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$. 4

6. (a) Show that the following statements are equivalent for a connected graph G: 3
   (i) G is Eulerian
   (ii) Every point of G has even degree
   (iii) The set of lines of G be partitioned into cycles
   (b) Draw one Euler graph each which contains: 2×2=4
   (i) Even number of vertices and odd number of edges
   (ii) Odd number of vertices and even number of edges

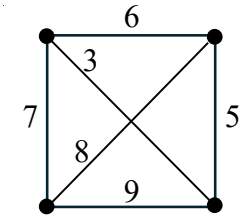
UNIT–IV

7. (a) Give an example of a graph which is: 2
   (i) Hamiltonian but not Eulerian
   (ii) Eulerian but not Hamiltonian
   (b) Find three distinct Hamiltonian cycles in the graph. Also find their weights. 3



   (c) Define maximal non-Hamiltonian graph. Give an example. 1+1=2

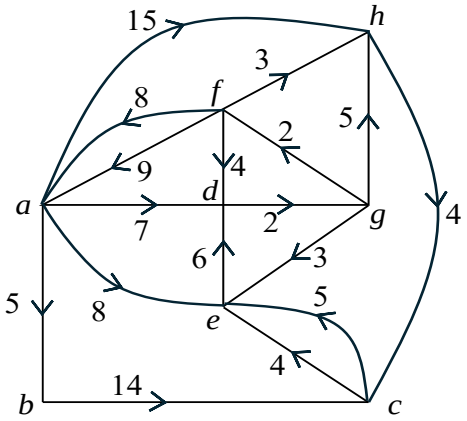
8. (a) Solve the travelling salesman problem. 4



   (b) Prove that the complete graph K_n, $n \geq 3$ is a Hamiltonian graph. 3

UNIT–V

9. Define the shortest path problem. Determine the shortest path between the vertices *a* to *g* using Dijkstra's algorithm: 1+6=7



10. Obtain the shortest distance matrix between all the vertices using Floyd Warshall algorithm: 7