2024

B.A./B.Sc.

Fourth Semester

GENERIC ELECTIVE – 4

MATHEMATICS

Course Code: MAG 4.11

(Differential Equations & Higher Trigonometry)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Solve $\frac{dy}{dx} = \frac{x+2y+1}{2x+4y+3}$ by reducing it to homogeneous form. 5

   (b) Solve $dx - (xy + x^2 y^3)dy = 0$. 5

   (c) Solve $(D^2 + 4D + 3)y = e^{-3x}$. 4

2. (a) Solve $(D^2 + 9)y = x \sin x$. 5

   (b) Solve $(D^4 - 2D^3 + D^2)y = 0$. 4

   (c) Solve $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$. 5

UNIT–II

3. (a) Find the integrating factor and solve the equation

   $ydx + (x^2 y - x)dy = 0$. 5

   (b) Solve $xp^2 + (y-x)p - y = 0$, where $p = \frac{dy}{dx}$. 4

   (c) Find the general and singular solution for $y = 3px + 6p^2 y^2$, where

   $p = \frac{dy}{dx}$. 5

4. (a) Find the integrating factor and solve the equation

$$(3x+2y^2)dx+2xydy=0.$$ 5

(b) Find the general and singular solution of $y = px+3p^2$, where

$$p=\frac{dy}{dx}.$$ 4

(c) Solve $y=2px+y^2p^3$, where $p=\frac{dy}{dx}$. 5

UNIT–III

5. (a) Solve by variation of parameters $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=3e^{-2x}$. 4

(b) Verify for integrability and solve by treating one variable as constant

$$2yzdx+zxdy-xy(1+z\ )dz\ =\ 0.$$ 5

(c) Solve:

$$x_1+2y_1-2x+2y=3e^t$$

$$3x_1+y_1+2x+y=4e^{2t}$$

where $x_1=\frac{dx}{dt}$, $y_1=\frac{dy}{dt}$. 5

6. (a) Solve by changing the independent variable

$$x\frac{d^2y}{dx^2}-2\frac{dy}{dx}+x^5y=2x^5.$$ 5

(b) Solve $\frac{dx}{x(y^2-z^2)}=\frac{dy}{y(z^2-x^2)}=\frac{dz}{z(x^2-y^2)}$. 5

(c) Check integrability and solve

$$(yz+xyz)dx+(zx+xyz)dy+(xy+xyz)dz=0$$ 4

UNIT–IV

7. (a) If $2\cos\theta = x+\frac{1}{x}$, $2\cos\phi = y+\frac{1}{y}$. Prove that one of the values of

$x^m y^n + \frac{1}{x^m y^n}$ is $2\cos(m\theta+n\phi)$. 5

(b) Find the value of $\frac{3\sin\theta-\sin 3\theta}{\theta(\cos\theta-\cos 3\theta)}$ where θ is zero. 4

(c) Form the equation whose roots are

$\cos\frac{\pi}{11}, \cos\frac{3\pi}{11}, \cos\frac{5\pi}{11}, \cos\frac{7\pi}{11}, \cos\frac{9\pi}{11}$

and find $\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11}$. 5

8. (a) If $x_r = \cos\frac{\pi}{2^r}+i\sin\frac{\pi}{2^r}$. Prove that $x_1x_2x_3\ldots\infty = -1$. 4

(b) Expand $\sin^8\theta$ in a series of cosines of multiples of θ. 5

(c) Solve the equation $x^{12}-1=0$ to find which of its roots satisfy $x^4+x^2+1=0$. 5

UNIT–V

9. (a) If $u = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$. Prove that $\tanh\frac{u}{2} = \tan\frac{\theta}{2}$. 4

(b) Prove that

$$\log\cos(x+iy) = \frac{1}{2}\log_e\left(\frac{\cosh 2y+\cos 2x}{2}\right) - i\tan^{-1}(\tan x\tanh y).$$

5

(c) Prove that $1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\ldots\infty = \frac{\pi}{2\sqrt{2}}$. 5

10. (a) If $\tan(\alpha + i\beta) = x + iy$. Show that $x \cot 2\alpha + y \coth 2\beta = 1$. 5

(b) Find the sum of the series

$$a \sin \alpha - \frac{1}{2} a^2 \sin 2\alpha + \frac{1}{3} a^3 \sin 3\alpha - \ldots \infty .$$ 4

(c) Separate into real and imaginary parts of $\sin^{-1}(\cos\theta + i \sin\theta)$. 5