2024
B.A./B.Sc.
Second Semester
GENERIC ELECTIVE – 2
MATHEMATICS
Course Code: MAG 2.11
(Algebra)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Give one example each of a sequence which is 4
   (i) bounded above but not below
   (ii) bounded below but not above
   (iii) bounded but not convergent
   (iv) not monotonic

   (b) Find the suitable positive integer m such that 4

   $$\left|\frac{2n+5}{6n-11}-\frac{1}{3}\right|<0.001, \forall n \geq m.$$

   (c) Suppose that $\{f_n\}$ and $\{\phi_n\}$ are two sequences such that $\lim_{n\to\infty} f_n = l$ and $\lim_{n\to\infty}\phi_n = k$ then prove that $\lim_{n\to\infty}\left(f_n\phi_n\right) = lk$. 6

2. (a) Show that the sequence $\{f_n\}$ where $f_n = \frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n}$ is bounded, monotonic, and converges to $\frac{1}{2}$. 6

   (b) Show that the sequence $\left\{\frac{3+2\sqrt{n}}{\sqrt{n}}\right\}$ converges to 2. 4

(c) Which of the following sets are bounded above, bounded below or otherwise? Also find the supremum and infimum, if they exist. Which of these belong to the set? 4

(i) $\left\{1+(-1)^n \frac{1}{n} : n \in \mathbb{N}\right\}$

(ii) $\left\{4+\frac{3}{n} : n \in \mathbb{N}\right\}$

UNIT–II

3. (a) Find the n^{th} term of the series $\sqrt{\frac{1}{4}}+\sqrt{\frac{2}{6}}+\sqrt{\frac{3}{8}}+\ldots$ and show that the series does not converge. 4

(b) Apply Cauchy integral test to examine the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}.$$ 5

(c) Prove that the positive term infinite geometrical series $1+r+r^2+r^3+\ldots+r^n+\ldots$, converges for $r<1$ and diverges for $r \geq 1$. 5

4. (a) Using Cauchy general principle of convergence for series, show that

$\sum_{n=1}^{\infty} \frac{1}{n}$ does not converge. 3

(b) Test the convergence or divergence of the series

$x+\frac{1.x^3}{2.3}+\frac{1.3.x^5}{2.4.6.7.}+\ldots, x>0$ using ratio test. 5

(c) Show that the series $1-\frac{1}{2^p}+\frac{1}{3^p}-\frac{1}{4^p}+\ldots$ converges for $p>0$. 4

(d) Is the series $\sum \frac{n}{n+1}$ convergent? Justify. 2

UNIT–III

5. (a) Find the quotient and remainder by synthetic division when the polynomial $3x^4 - 4x^3 + 2x^2 - 9x + 1$ is divided by $2x + 1$. 4

   (b) Solve the equation $x^3 - 6x^2 - 6x - 7 = 0$ by Cardan's method. 6

   (c) If α, β, γ be the roots of the equation $x^3 + px + q = 0$, express in terms of p and q the values of the symmetric functions: 4

   (i) $\sum \alpha^2$

   (ii) $\sum \alpha^3$

6. (a) Solve the equation $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$ given that one of its root is $-2+\sqrt{3}$. 4

   (b) Given that the roots of the equation $x^3 - 3x^2 + kx + 3 = 0$ are in arithmetic progression. Find the value of k and solve the equation. 5

   (c) Find the equation whose roots are the squared differences of the cubic equation $x^3 + px + q = 0$. 5

UNIT–IV

7. (a) Show that the operation $*$ on $\mathbb{Q}-\{1\}$, define by $a*b = a+b-ab$, for all $a,b \in \mathbb{Q}-\{1\}$ satisfies: 5

   (i) The closure property

   (ii) The associative law

   (iii) The commutative law

   (iv) What is the identity element?

   (v) For each $a \in \mathbb{Q}-\{1\}$, find the inverse of a.

   (b) Define subgroup of a group and prove that the intersection of two subgroups of a group G is also a subgroup of G. 4

   (c) Define cyclic group and prove that every cyclic group is abelian. Is the converse true? Justify. 5

8. (a) Show that the set $\{1, -1, i, -i\}$ forms a group under multiplication. 5

(b) Show that the set of all 2×2 real matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with $ad - bc = 1$ is a subgroup of the group of all 2×2 non-singular real matrices under matrix multiplication. 5

(c) If $S = \{2, 4, 6, 8\}$. Draw up a multiplication table modulo 10 and show that S is a group under multiplication modulo 10. 4

UNIT–V

9. (a) Write down the elements of the symmetric group S_3 for the set of 3 integers $\{1, 2, 3\}$. Make the multiplication table for S_3. 4

(b) State and prove Lagrange's theorem for finite groups. Is the converse true? Justify. 6

(c) Prove that every group of prime order is cyclic. 4

10. (a) If G is a finite group of order n, prove that G has $\phi(n)$ generators, where ϕ is the Euler's phi function. 5

(b) Define even and odd permutation. Are the following permutations even or odd? 4

(i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 2 & 1 & 6 & 4 \end{pmatrix}$

(ii) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$

(c) If G is a group and $a \in G$ is of order n, show that if $a^k = e$, then n divides k. 3

(d) Give an example of a group of order 4 which is not cyclic. 2