2024

B.A./B.Sc.

Sixth Semester

DISCIPLINE SPECIFIC ELECTIVE – 3

MATHEMATICS

Course Code: MAD 6.11

(Theory of Equations)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Express $x^5 + 5x^3 + 3x = 0$ as a polynomial in $(x-1)$. Also, find $f(x+1)$. 4

   (b) Prove that every equation of degree n has exactly n roots. 6

   (c) Find the equation whose roots are $4-\sqrt{2}, 2+i\sqrt{3}$. 4

2. (a) Apply Descartes' rule of signs to discuss the nature of the roots of the equation $3x^4 + 12x^2 + 5x - 4 = 0$. 4

   (b) Show that the roots of the equation $\frac{1}{x-a}+\frac{1}{x-b}+\frac{1}{x-c}=\frac{1}{x}$, where $a > b > c > 0$ are real. 5

   (c) Find the condition that the roots $\alpha, \beta, \gamma, \delta$ of the biquadratic $x^4 + px^3 + qx^2 + rx + s = 0$ should have its roots connected by the relation $\beta + \gamma = \alpha + \delta$. 5

UNIT–II

3. (a) If a, b, c be the roots of the equation $x^3 - px^2 + qx - r = 0$, find the value of: 4

   (i) $\sum\left(\frac{b}{c}+\frac{c}{b}\right)$ (ii) $\sum a^2b^2$

(b) Remove the fractional coefficient of the equation

$$x^3 - \frac{5}{2}x^2 - \frac{7}{18}x + \frac{1}{108} = 0.$$ 4

(c) Find the equation whose roots are the cubes of the roots of the equation $x^3 + ax^2 + bx + c = 0$. 6

4. (a) Remove the third term of the equation $x^4 - 4x^3 - 18x^2 - 3x + 2 = 0$ and solve the equation. 4

(b) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then: 6

(i) Form the equation whose roots are $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}, \gamma + \frac{1}{\gamma}$

(ii) Find the value $\left(\frac{1}{\beta} + \frac{1}{\gamma} - \frac{1}{\alpha}\right) \times \left(\frac{1}{\gamma} + \frac{1}{\alpha} - \frac{1}{\beta}\right) \times \left(\frac{1}{\alpha} + \frac{1}{\beta} - \frac{1}{\gamma}\right)$

(c) If α be an imaginary root of $x^n - 1$, where n is a prime number, prove that $(1 - \alpha)(1 - \alpha^2) \ldots \ldots \ldots (1 - \alpha^{n-1}) = n$. 4

UNIT–III

5. (a) Solve $x^3 + 6x^2 - 12x + 32 = 0$ by Cardan's method. 5

(b) Show that $2 \sin 10°, 2 \sin 50°, -2 \sin 70°$ are the roots of the equation $x^3 - 3x + 1 = 0$. 5

(c) Find the Euler's cubic of the equation $x^4 - 2x^2 + 8x - 3 = 0$ and hence solve it. 4

6. (a) Apply Descartes' method to solve the biquadratic equation $x^4 + 6x^3 + 14x^2 + 22x + 5 = 0$. 7

(b) Solve the equation $x^4 - 12x^3 + 41x^2 - 18x - 72 = 0$ by Ferrari's method. 7

UNIT–IV

7. (a) If $\alpha, \beta, \gamma, \delta$ be the roots of the equation $x^4 + px^2 + qx + r = 0$, then show that $\sum \alpha^6 = 6pr + 3q^2 - 2p^3$ and $\sum \alpha^7 = -7q\left(p^2 - r\right)$. 6

(b) Prove that if α_1 be the root of the equation $f(x) = 0$ of multiplicity r, then α_1 is a root of the equation $f'(x) = 0$ of multiplicity $(r-1)$, where $f'(x)$ is the first derived function of $f(x)$. 4

(c) Find the multiple roots of the equation $x^4 + 3x^3 - 7x^2 - 15x + 18 = 0$. 4

8. (a) Show that (-8) is a superior limit of the negative roots of the equation $x^4 - 2x^3 - 13x^2 + 38x - 24 = 0$. 4

(b) Find the consecutive integers which contain the real roots of the equation $x^4 - x^3 - 4x^2 + 4x + 1 = 0$. 5

(c) Analyze the characters of the roots of the equation $x^5 + x^4 + x^2 - 25x - 36 = 0$. 5

UNIT–V

9. (a) Use Sturm's theorem to show that the equation $x^3 - 7x + 7 = 0$ has one root between -4 and -3 and two roots between 1 and 2. 5

(b) Prove by Sturm's method that the equation $x^4 - 6x^3 + 13x^2 - 12x + 4 = 0$ has two pairs of equal roots. 5

(c) Find the condition that the roots of the equation $ax^2 + 2bx + c = 0$ should be real. 4

10. (a) Using Newton's method of approximation, find the positive roots of the equation $x^3 + x^2 + x - 100 = 0$ correct to two places of decimals. 4

(b) Find by Horner's method the real roots of the equation $x^3 + 3x - 7 = 0$ correct to two places of decimal. 5

(c) Find the positive roots of the equation $x^4 - 12x + 7$ correct to 7 places of decimals. 5
