2024

B.A./B.Sc.

Fourth Semester

CORE – 8

MATHEMATICS

Course Code: MAC 4.11

(Numerical Methods)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Design an algorithm to determine if a positive integer is prime or not. 7

   (b) A rectangular board is measured with a scale having accuracy of 0.2 cm. The length and breadth are measured as 35.4 cm and 18.4 cm respectively. Find the relative error and the percentage error in the area calculated. 5

   (c) Explain round off error with example. 2

2. (a) Define an algorithm. Design an algorithm to determine the greatest of three numbers. 7

   (b) Find the sum of the numbers 0.1532, 15.45, 0.000354, 305.1, 8.12, 143.3, 0.0212, 0.643 and 0.1734, where each number is correct to the digits given. Estimate the absolute error in the sum. 5

   (c) Find the absolute error and relative error in $\sqrt{6}+\sqrt{7}+\sqrt{8}$ correct to 4 significant digits. 2

UNIT–II

3. (a) Define rate of convergence. Show that the Newton Raphson's method has second order convergence. 7

(b) Apply secant method to determine a root of the equation $f(x) = \cos x - xe^{x} = 0$ by taking the initial approximations $x_0 = 0$, $x_1 = 1$. Perform three iterations. 7

4. (a) Determine the initial approximation for finding the smallest positive root of the equation $x - e^{-x} = 0$. Use these to find the root correct to three decimal places by Regula-Falsi method. 7

(b) Find the interval in which the smallest positive root of the equation $x^3 - x - 4 = 0$ lie. Determine the root using the bisection method. Perform four iterations. 7

UNIT–III

5. (a) Solve the following system of equations using the Gauss elimination method with partial pivoting: 5

$$x_1 + 10x_2 - x_3 = 3$$
$$2x_1 + 3x_2 + 20x_3 = 7$$
$$10x_1 - x_2 + 2x_3 = 4$$

(b) Prove that the Gauss Jacobi iteration method for the solution $Ax = b$ converges to the exact solution for any initial vector, if $\|H\| < 1$. 5

(c) Show that the matrix $\begin{bmatrix} 12 & 4 & -1 \\ 4 & 7 & 1 \\ -1 & 1 & 6 \end{bmatrix}$ is positive definite. 4

6. (a) Solve the system of equations $Ax = b$, where

$$A = \begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}.$$

Using the LU decomposition method. Take all the diagonal elements of L as 1. Also find A^{-1}. 8

(b) Solve the system of equations

$$45x_1 + 2x_2 + 3x_3 = 58$$
$$-3x_1 + 22x_2 + 2x_3 = 47$$
$$5x_1 + x_2 + 20x_3 = 67$$

Correct to three decimal places, using the Gauss-Seidel iteration method. Take the initial approximation as $x^{(0)} = 0$. 6

UNIT–IV

7. (a) Taking the interval of difference as unity, evaluate the following: (a and b are constants) 5

(i) $\Delta \tan(ax)$

(ii) $\Delta^n (e^{ax+b})$

(b) Given that $y_{35.0} = 1175$, $y_{35.5} = 1280$, $y_{39.5} = 2180$, $y_{40.5} = 2420$, obtain a value for y_{40}. 5

(c) Find the missing term in the following table: 4

x:	0	1	2	3	4
y:	1	3	9	-	81

8. (a) Establish the following relations: 5

(i) $\Delta \equiv \mu\delta + \dfrac{\delta^2}{2}$

(ii) $\left(\Delta - \dfrac{1}{2}\delta^2\right) = \delta\sqrt{1 + \dfrac{\delta^2}{4}}$

(b) For the following data, calculate the differences and obtain the Gregory-Newton forward difference polynomial. Interpolate at $x = 0.25$. 5

x:	0.1	0.2	0.3	0.4	0.5
$f(x)$:	1.40	1.56	1.76	2.00	2.28

(c) Using the Lagrange interpolation for the data: 4

x:	–1	1	4	7
$f(x)$:	–2	0	63	342

Interpolate at $x = 5$.

UNIT–V

9. (a) Using trapezoidal rule, evaluate the integral $I = \int_0^1 \frac{dx}{x^2 + 6x + 10}$ with four subintervals. 4

(b) Using Simpson's $\frac{1}{3}^{\text{rd}}$ rule, evaluate $\int_0^1 xe^x dx$ taking four intervals. compare the result with the actual value. 5

(c) Considering the initial value problem $y' = x(y + x) - 2$, $y(0) = 2$, use Euler's method with step size $h = 0.2$ to compute approximation to $y(0.6)$. 5

10. (a) A river is 80 feet wide. The depth d (in feet) of the river at a distance x from one bank is given by the following table:

x:	0	10	20	30	40	50	60	70	80
d:	0	4	7	9	12	15	14	8	3

Find approximately the area of the cross section of the river using these data. 7

(b) Use the Runge-Kutta formula of fourth order to find the numerical solution at $x = 0.8$ for $\frac{dy}{dx} = \sqrt{x + y}$, $y(0.4) = 0.41$.

Assume the step length $h = 0.2$ 7