2023

M.Sc.

Fourth Semester

DISCIPLINE SPECIFIC ELECTIVE – 04

MATHEMATICS

Course Code: MMAD 4.21

(Fluid Mechanics)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Derive the equation of continuity by Euler's method. 5

(b) Show that $u=-\frac{2xyz}{(x^2+y^2)^2}, v=\frac{(x^2-y^2)z}{(x^2+y^2)^2}, w=\frac{y}{x^2+y^2}$ are the velocity components of a possible liquid motion. Is this motion irrotational? 5

(c) Define stream lines and path lines. Derive the equations of stream lines and path lines. 2+2=4

2. (a) For a steady, inviscid and incompressible flow with negligible body forces, velocity components in cylindrical polar co-ordinates are given by $u_r=-U\left(1-\frac{a^2}{r^2}\right)\cos\theta,\ u_\theta=U\left(1+\frac{a^2}{r^2}\right)\sin\theta$ and $u_z=0$, show that it is a possible solution of momentum equations. (U and a are constants.) 7

(b) Derive the equation of energy $\frac{d}{dt}(T+W+I)=\int_s p\vec{q}.\vec{n}ds$, where symbols have their usual meanings. 7

UNIT–II

3. (a) Define stream function. Describe physical significance of stream function. 2+4=6

(b) Show that the velocity potential $\phi = \frac{1}{2} \ln \frac{(x+a)^2 + y^2}{(x-a)^2 + y^2}$ gives a possible motion; also show that the stream lines are circles.

4+4=8

4. (a) Find the complex potential due to a doublet. 6

(b) State and prove the theorem of Blasius. 8

UNIT–III

5. (a) Derive velocity potential, stream function, and complex potential due to a rectilinear vortex filament. 4+2+2=8

(b) Two infinite rows of vortices are placed parallel to each other at *a* distance *b* apart with upper row having vortices of strength *k* each and lower row having vortices of strength –*k* each. Find complex potential of the system and show that the vortex system moves parallel to itself with velocity $\frac{k}{2a} \coth\left(\frac{\pi b}{a}\right)$, where *a* is the distance between any two vortices in the upper row. 6

6. (a) An infinite row of equidistant rectilinear vortices are at a distance *a* apart. The vortices are of the same strength *k* but they are alternatively of opposite signs. Find velocity potential and the stream function of the system. 7

(b) Describe in detail the Karman Vortex street and find velocity components at the origin. 7

UNIT–IV

7. (a) The stress tensor at a point *P* is $\sigma_{ij} = \begin{pmatrix} 7 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{pmatrix}$. Determine the stress vector on the plane at *P* whose unit normal is $\vec{n} = (2/3)\hat{i} - (2/3)\hat{j} + (1/3)\hat{k}$ 4

(b) Show that the general motion of a fluid element can be expressed as the combination of translation, rotation and deformation of the fluid element. 7

(c) Define viscosity. State Newton's law of viscosity. Write the dimension of kinematic viscosity. 1+1+1=3

8. (a) Derive the vorticity transport equation. 7

(b) Discuss the plane Couette flow of a viscous fluid. 7

UNIT–V

9. (a) Derive the Bernoulli's equation for compressible flow undergoing isothermal and adiabatic processes. 3+6=9

(b) A 120 mm diameter pipe reduces to 60 mm diameter through a sudden contraction. When it carries air at 25° C under isothermal condition, the absolute pressure observed in the two pipes just before and after the contraction are 480 kN/m^2 and 384 kN/m^2 respectively. Find the densities at the two sections.
(Take R = 287 J/Kg-K) 5

10. (a) Derive the equation of motion of a gas in an isentropic flow process. 7

(b) Derive mass rate of flow of compressible fluid through a convergent nozzle. 7