2023
M.Sc.
Fourth Semester
CORE – 11
MATHEMATICS
Course Code: MMAC 4.11
(Mathematical Methods)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) If $\mathcal{L}\{F(t)\} = f(s)$, then prove that $\mathcal{L}\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s)$ where $n = 1, 2, 3, ...$ 5

   (b) Show that $\int_0^\infty \frac{e^{-t}\sin t}{t} dt = \frac{\pi}{4}$. 5

   (c) Evaluate using convolution theorem $\mathcal{L}^{-1}\left\{\frac{1}{(s+3)(s-1)}\right\}$ 4

2. (a) Solve the simultaneous ordinary differential equation using Laplace transform $\left.\begin{aligned} Y' + Z'' &= t \\ Y'' - Z &= e^{-t} \end{aligned}\right\}$ subject to the conditions $Y(0) = 3$, $Y'(0) = -2$, $Z(0) = 0$ 5

   (b) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & \text{if } |x| \le 1 \\ 0, & \text{if } |x| > 1 \end{cases}$ 4

   (c) Using Parseval's identity, prove that $\int_0^\infty \frac{x^2}{(x^2+1)^2} dx = \frac{\pi}{4}$ 5

UNIT–II

3. (a) Verify that $\varphi(x)=\sin\left(\frac{\pi x}{2}\right)$ is a solution of the integral equation

$$\varphi(x)=\frac{x}{2}+\frac{\pi^2}{4}\int_0^1 k(x,t)\,\varphi(t)dt \text{ where}$$

$$k(x,t)=\begin{cases}\frac{1}{2}x(2-t), & 0\le x\le t\\ \frac{1}{2}t(2-x), & t\le x\le 1\end{cases}$$ 4

(b) Obtain Fredholm integral equation corresponding to the boundary value problems $\frac{d^2Y}{dx^2}+\lambda Y=x:Y(0)=0,Y'(1)=0$ 4

(c) Solve the integral equation $\varphi(x)=1+x+\int_0^x (x-t)\varphi(t)dt$ by successive approximation method. 6

4. (a) Solve the Fredholm integral equation $\varphi(x)=x^2-x^4+2\int_0^x 4t\varphi(t)dt$ by using the method of successive substitution. 4

(b) Solve the integral equation: 5×2=10

(i) $\varphi(x)-\lambda\int_{-\pi}^{\pi}(x\cos t+t^2\sin x+\cos x\sin t)\varphi(t)dt=x$

(ii) $\varphi(x)-\lambda\int_0^{\frac{\pi}{2}}\sin x\cos t\varphi(t)dt=\sin x$

UNIT–III

5. (a) Using Laplace transform solve the integral equation

$$\varphi(x)=\cos x-x-2+\int_0^x (t-x)\varphi(t)\,dt$$ 5

(b) Find the resolvent kernel using Fredholm determinant $k(x,t)=x^2t-xt^2$ and hence solve the integral equation

$$\varphi(x)-\lambda\int_0^1\left(x^2t-xt^2\right)\varphi(t)dt=1$$ 5

(c) Show that the integral equation $\varphi(x)-\lambda\int_0^{2\pi}\sin x\cos t\,\varphi(t)dt=0$ has no eigen value and eigen function. 4

6. (a) Find the characteristic number and eigen function of the homogenous integral equation $\varphi(x)-\lambda\int_0^{\pi}K(x,t)\varphi(t)dt=0$, where

$$K(x,t)=\begin{cases} x(t-1), & 0\le x\le t \\ t(x-1), & t\le x\le 1 \end{cases}$$ 7

(b) In a homogeneous Fredholm integral equation with symmetric kernel, prove that every pair of eigen function corresponding to different eigen values is orthogonal. 7

UNIT–IV

7. (a) Prove that $\left(1+\{4t\}\right)*\left(1+2\{\cos 2t-\sin 2t\}\right)=1+\{2\}$ 5

(b) Show that $l^3\left\{n^2te^{-nt}\right\}$, where $l=$ integral operator. 3

(c) Calculate the value of $\frac{5s+3}{(s-1)(s^2+2s+5)}$, where $s=$ differential operator. 3

(d) Show that $\frac{\{e^t-\sin t-\cos t\}}{\{\sin t\}}=\{2e^t\}$ 3

8. (a) Prove that 3×2=6

(i) $\delta(x-a)=\delta(a-x)$

(ii) $f(x)\delta(x-a)=f(a)\delta(x-a)$

(b) Show that $\lim_{a\to 0}\left(\frac{\sin 2\pi ax}{\pi a}\right)$ is a Dirac delta function. 4

(c) Find the Fourier and Laplace transforms of Dirac delta function 4

UNIT–V

9. (a) Define a regular Sturm-Liouville problem. Rewrite the Bessel's equation $x^2y''+xy'+(\lambda x^2-n^2)y=0$ in Sturm-Liouville form.

1+2=3

(b) Find the eigen value and function of the Sturm-Liouville problem 5

$$y'' + \lambda y = 0, 0 < x < L$$

$$y(0) = 0, y(L) = 0$$

(c) Express the function $f(x)=1$ as the eigen function series of the given Sturm-Liouville problem. 6

$$y'' + \lambda y = 0, 0 < x < L$$

$$y(0) = 0, hy(L) + y'(L) = 0, h > 0$$

10. (a) Find the Green's function for the boundary value problem 7

$$y'' - k^2 y = 0, k \neq 0$$

$$y(0) = y(1) = 0$$

(b) Using Green's function solve the boundary value problem 7

$$y'' + y = x$$

$$y(0) = y\left(\frac{\pi}{2}\right) = 0$$