2023

M.Sc.

Second Semester

CORE – 05

MATHEMATICS

Course Code: MMAC 2.11

(Partial Differential Equations)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Form the partial differential equation by eliminating the arbitrary function ϕ from $\phi(xy+z^2, x+y+z)=0$. 3

(b) Find the integral surface of the partial differential equation $(x-y)p+(y-x-z)q=z$ through the circle $x^2+y^2=1, z=1$. 5

(c) Find the surface which is orthogonal to the system $z=cxy(x^2+y^2)$ and passes through the hyperbola $x^2-y^2=a^2, z=0$. 6

2. (a) Find the characteristics of the partial differential equation $pq=z$ and determine the integral surface which passes through the straight line $x=1, z=y$. 5

(b) Show that partial differential equations $xp=yq$ and $z(xp+yq)=2xy$ are compatible and hence find their solution. 4

(c) Find the complete integral of the partial differential equation $x^2p^2+y^2q^2=4$ by Charpit's method. 5

UNIT–II

3. (a) Classify and reduce the partial differential equation $x^2u_{xx}+2xyu_{xy}+y^2u_{yy}=0$ to a canonical form and hence solve it. 1+4+1=6

(b) Prove that, for the partial differential equation $u_{xy} + \frac{u}{4} = 0$, the Green function is $v(x, y; \xi, \eta) = J_0\sqrt{(x-\xi)(y-\eta)}$ where J_0 denotes Bessel's function of the first kind of order zero. 8

4. (a) If $L(u) = a(x)u_{xx} + b(x)u_x + c(x)u$ then construct its adjoint $L^*(v)$. 4

(b) Find the complete solution of the partial differential equation $\left(x^2D^2 + 2xyDD' + y^2D'^2\right)u = x^2y^2$. 5

(c) Solve the partial differential equation $(q+1)s = (p+1)t$ by Monge's method. 5

UNIT–III

5. (a) Derive the Poisson equation. 6

(b) Show that the 2D-Laplace equation in polar co-ordinates can be written as $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$. 8

6. (a) Define harmonic function. State and prove the mean value theorem for harmonic functions. 6

(b) Using the method of separation of variables, discuss the solution of Dirichlet problem for a rectangle. 8

UNIT–IV

7. (a) Show that the function $u(x,t) = \frac{1}{\sqrt{4\pi\alpha t}} e^{\frac{-(x-\xi)^2}{4\alpha t}}$ is a solution of the diffusion equation $u_t = \alpha u_{xx}$. 7

(b) Solve the following initial boundary value problem by using the method of separation of variables: 7

$$u_t = \alpha u_{xx},\ 0 < x < 1,\ t > 0$$
$$u(0,t) = 2,\ u(1,t) = 3$$
$$u(x,0) = x(1-x)$$

8. (a) A bar 10 cm long with insulated sides, has its ends *A* and *B* kept at 20 °C and 40 °C respectively unless steady state condition prevail. The temperature at *A* is then suddenly raised to 50 °C and at the same instant that at *B* is lowered to 10 °C. Find the subsequent temperature at any point on the bar at any time. 7

(b) Discuss the exact solution of the initial value problem for one-dimension Burger's equation. 7

UNIT–V

9. (a) Find the solution of the one-dimensional non-homogeneous wave equation: $u_{tt} - c^2 u_{xx} = f(x,t), x \in \mathbb{R}, t > 0$ subject to initial conditions $u(x,0) = \phi(x), u_t(x,0) = \psi(x)$. 8

(b) Obtain the solution of the wave equation $u_{tt} = c^2 u_{xx}$ subject to the following conditions: 6

$u(0,t) = u(2,t) = 0;$

$u(x,0) = \sin^3\left(\frac{\pi x}{2}\right);$

$u_t(x,0) = 0;$

10. (a) Discuss the solution of the problem of vibrating string using variables separable method. 7

(b) Derive the wave equation representing the transverse vibration of a string in the form: 7

$$u_{tt} = c^2 \left\{1 + (u_x)^2\right\}^{-2} u_{xx}$$