2023

B.A./B.Sc.

Second Semester

GENERIC ELECTIVE – 2

STATISTICS

Course Code: STG 2.11

(Introductory probability)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) If X and Y are two random variables with variances σ_X^2 and σ_Y^2 respectively and r is the coefficient of correlation between them. If $U = X + KY$ and $V = X + \left(\frac{\sigma_X}{\sigma_Y}\right)Y$, find the value of K so that U and V are uncorrelated. 7

   (b) The random variables X and Y are jointly normally distributed and U and V are defined by $U = X\cos\alpha + Y\sin\alpha$ and $V = Y\cos\alpha - X\sin\alpha$. Show that U and V will be uncorrelated if $\tan 2\alpha = \frac{2r\sigma_X\sigma_Y}{\sigma_X^2 - \sigma_Y^2}$. 4

   (c) Fit a second degree equation $Y = a + bx + cx^2$ by the method of least square. 3

2. (a) Show that the coefficient of correlation r is independent of change of scale and origin of the variable. Also prove that two independent variables are uncorrelated. 5+2=7

   (b) Fit an exponential curve $Y = ab^X$ by the method of least square. 4

   (c) Define regression analysis. Prove that if one of the regression coefficients is greater than unity, the other must be less than unity. 1+2=3

UNIT–II

3. (a) Define partial and multiple correlation. Write the properties of multiple correlation coefficients. 2+3=5

   (b) Show that the partial correlation coefficient between X_1 and X_2 is

$$r_{12.3} = \frac{\operatorname{cov}(X_{1.3}, X_{2.3})}{\sqrt{\operatorname{var}(X_{1.3})\operatorname{var}(X_{2.3})}}.$$ 6

   (c) Explain two properties of residuals. 1½+1½=3

4. (a) In the usual notations, prove that

$$R^2_{1.23} = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{23}r_{31}}{1 - r_{23}^2} \le r_{12}^2$$ 6

   (b) If r_{12} and r_{13} are given, show that

$$r_{12}r_{13} - \sqrt{(1 - r_{12}^2 - r_{13}^2 + r_{12}^2 r_{13}^2)} \le r_{12} \le r_{12}r_{13} + \sqrt{(1 - r_{12}^2 - r_{13}^2 + r_{12}^2 r_{13}^2)}$$

   And if $r_{12} = k$ and $r_{13} = -k$, show that r_{23} will lie between –1 and $1-2k^2$. 4+4=8

UNIT–III

5. (a) A random variable X has the following probability function:

Value of X, x:	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

   (i) Find k

   (ii) Evaluate $P(X < 6)$, $P(X \ge 6)$ and $P(0 < X < 5)$

   (iii) If $P(X \le a) > \frac{1}{2}$, find the minimum value of a

   (iv) Determine the distribution function of X. 1+2+1+2=6

   (b) Define random variable. Discuss different types of random variables with examples. 1+3=4

   (c) In four tosses of a coin, let X be the number of heads. Tabulate the 16 possible outcomes with the corresponding values of X. By simple counting, derive the probability distribution of X and hence calculate the expected value of X. 4

6. (a) Define the following: 1½+1½=3
   (i) Convergence in probability
   (ii) Moment generating functions
   (b) Show that the mathematical expectation of the sum of two random variable is the sum of their individual expectations. 2
   (c) A coin is tossed until a head appears. What is the expectation of the number of tosses required? 4
   (d) State and prove Chebyshev's inequality. 1+4=5

UNIT–IV

7. (a) Derive the Poisson distribution as a limiting case of binomial distribution. 6
   (b) A coffee connoisseur claims that he can distinguish between a cup of instant coffee and a cup of percolator coffee 75% of the time. It is agreed that hid claim will be accepted if he correctly identifies at least 5 of the 6 cups. Find his chances of having the claim
   (i) accepted
   (ii) rejected, when he does have the ability he claims 4
   (c) Obtain the mean and variance of the hyper geometric distribution. 4

8. (a) Determine the binomial distribution for which the mean is 4 and the variance is 3 and find its mode. 3
   (b) Define hyper geometric distribution. Obtain the binomial distribution as a limiting case of hyper geometric distribution. 1+3=4
   (c) Obtain the moment generating function of geometric distribution. 3
   (d) If X and Y are independent Poisson variates such that $P(X=1)=P(X=2)$ and $P(Y=2)=P(Y=3)$. Find the variance of $X-2Y$. 4

UNIT–V

9. (a) Define exponential distribution. Hence, obtain its mean, variance, and m.g.f. 1+4=5

(b) Find the normal distribution as a limiting form of binomial distribution. Prove that for normal distribution, the quartile deviation, the mean deviation and the standard deviation are approximately 10:12:15.

4+5=9

10. (a) Define rectangular distribution. Hence obtain its mean, variance and m.g.f. 1+4=5

(b) If X is uniformly distributed with mean 1 and variance $\frac{4}{3}$, find $P(X < 0)$. 3

(c) Obtain the mean, variance and m.g.f. of the normal distribution.

1+1+4=6
