2023

B.A./B.Sc.

Fourth Semester

CORE – 8

STATISTICS

Course Code: STC 4.11

(Statistical Inference)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) What is an estimator? When would you say that estimate of a parameter is good? 2+1=3

(b) Define unbiased estimator and minimum variance unbiased estimator. 2+2=4

(c) If $x_1, x_2, ..., x_n$ is a random sample from a normal population $N(\mu, 1)$. Show that $t = \frac{1}{n}\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of $\mu^2 + 1$. 3

(d) Let T_1 and T_2 be two unbiased estimates of $\gamma(\theta)$ with variances σ_1^2, σ_2^2 and correlation ρ. What is the best unbiased linear combination of T_1 and T_2 and what is the variance of such a combination? 4

2. (a) Define an efficient estimator. 2

(b) Let T_1 and T_2 be two unbiased estimates of $\gamma(\theta)$ with efficiencies e_1 and e_2 respectively and $\rho = \rho_\theta$ be the correlation coefficient between them, then

$$\sqrt{e_1 e_2} - \sqrt{(1-e_1)(1-e_2)} \le \rho \le \sqrt{e_1 e_2} + \sqrt{(1-e_1)(1-e_2)}$$ 5

(c) X_1, X_2 and X_3 is a random sample of size 3 from a population with mean value μ and variance σ^2. T_1, T_2, T_3 are the estimators used to estimate mean value μ where

$$T_1 = X_1 + X_2 - X_3$$
$$T_2 = 2X_1 + 3X_3 - 4X_2$$

and $T_3 = \frac{1}{3}(\lambda X_1 + X_2 + X_3)$

(i) Are T_1 and T_2 unbiased estimators?

(ii) Find the value of λ such that T_3 is unbiased estimator for μ.

(iii) With this value of λ is T_3 a consistent estimator?

(iv) Which is the best estimator? 2+2+1+2=7

UNIT–II

3. (a) State and prove Cramer-Rao inequality. 2+5=7

(b) Define confidence interval and confidence limits. Obtain $100(1-\alpha)\%$ confidence limits for the parameter λ of the Poisson distribution

$$f(x,\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}; x = 0,1,2,\ldots.$$ 2+5=7

4. (a) Let X and Y be a random variables such that $E(Y) = \mu$ and $Var(Y) = \sigma_Y^2 > 0$. Let $E(Y/X = x)$. Then show that

(i) $E[\varnothing(X)] = \mu$

(ii) $Var[\varnothing(X)] \leq Var(Y)$ 3+4=7

(b) Define MVB. Show that $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, in a random sampling from an exponential population having p.d.f. $f(x,\theta) = \frac{1}{\theta}e^{\frac{-x}{\theta}}; 0 < x < \infty$, where $0 < \theta < \infty$, in an MVB estimator of θ and has variance $\frac{\theta^2}{n}$.

2+5=7

UNIT–III

5. (a) Find the M.L.E. for the parameter λ of a Poisson distribution on the basis of a sample of size n. 4

(b) Describe the method of moments for estimating the parameters. What are the properties of the estimates obtained by this method? 3+2=5

(c) Find the Bayes' estimator of the parameter p of a binomial distribution with x successes out of n given that the prior distribution of p is a beta distribution with parameters α and β. 5

6. (a) Prove that the M.L.E. of the parameter α of a population having density function $\frac{2}{\alpha^2}(\alpha - x); 0 < x < \alpha$. For a sample of unit is $2x$, x being the sample value. Also show that the estimate is biased. 5

(b) Estimate α and β in the case of Pearson's type III distribution

$$f(x;\alpha,\beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, 0 \leq x < \infty$$ 5

(c) What modification in minimum chi-square method of estimation gives rise to the method of modified minimum chi-square? What properties the minimum chi-square estimators hold? 4

UNIT–IV

7. (a) Define most powerful test? 2

(b) Let p be the probability that a coin will fall head in a single test in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find α and $1-\beta$. 5

(c) Describe likelihood ratio test and state its important properties. 5+2=7

8. (a) Define unbiased test and unbiased critical region with usual notations. 2

(b) Use the Neyman-Pearson Lemma to obtain the region for testing $\theta = \theta_0$ against $\theta = \theta_1 > \theta_0$ and $\theta = \theta_1 < \theta_0$, in the case of a normal population $N(\theta, \sigma^2)$, where σ^2 is known. 7

(c) Explain the test for the equality of means of two normal populations when population variances are unequal. 5

UNIT–V

9. (a) What is distribution free test? When should the non-parametric method be preferably used? 2+2=4

   (b) Explicate sign test for testing paired samples? 5

   (c) Stating the underlying assumptions and the null hypothesis, develop the Wilcoxon test. 5

10. (a) Explain the main difference between parametric and non-parametric approaches to the theory of statistical inference. 5

    (b) Describe Wald-Wolfowitz run test for testing an identicalness of two populations. 5

    (c) How can one use the Kolmogorov-Smirnov test for two sample problem? 4