2023
B.A./B.Sc.
Second Semester
CORE – 4
STATISTICS
Course Code: STC 2.21
(Algebra)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) What is the fundamental theorem of algebra? Explain with examples. 2+3=5
   (b) Solve the equation $2x^3 - x^2 - 22x - 24 = 0$ given that two roots are in the ratio 3:4. 4
   (c) Prove the relation between roots and coefficients of a polynomial equations of degree *n*. 5

2. (a) Given that $-2 + i\sqrt{7}$ is a roots of the equation $x^4 + 2x^2 + 16x + 77 = 0$. Solve it completely. 5
   (b) Show that the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ are in arithmetic progression if $p^3 - 4pq + 8r = 0$. 5
   (c) Explain linear dependence and independence with examples. 2+2=4

UNIT–II

3. (a) If *A*, *B*, *C* are three matrices such that

$$A = \begin{bmatrix} x & y & z \end{bmatrix}, B = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}, C = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$, find *ABC*. 5

   (b) Define idempotent and involutory matrix with examples. 4

(c) Define adjoint of a matrix. If A be any n-rowed square matrix, then prove that $A.(Adj\,A) = (Adj\,A).A = |A|.I_n$
where, I_n is any n-rowed identity matrix. 5

4. (a) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that $A^2 - 4A - 5I = 0$, where, I is a unit matrix of order 3. 4

(b) Define symmetric and skew symmetric matrix. Show that every square matrix is uniquely expressible as the sum of a symmetric and skew symmetric matrix. 2+4=6

(c) Define inverse of a matrix. If A and B are non singular matrices of order n, then show that $(AB)^{-1} = B^{-1}A^{-1}$ 1+3=4

UNIT–III

5. (a) What is a minor of a determinant? Express the cofactors of a determinant in terms of minors. 2

(b) Show that in a determinant the sum of the products of the elements of any row or column with the cofactors of the corresponding elements of any other row (or columns) is zero. 4

(c) If a, b, c are all different and if $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$,
prove that $abc = -1$. 4

(d) Solve the following system of linear equations with the help of Cramer's rule:

$$x + y + z = 12$$
$$x + 2y + z = 16$$
$$2x - y + z = 14$$
4

6. (a) Show that $|A| = 0$, where, $A = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$ 2

(b) Prove that $\Delta = \begin{bmatrix} a^3 & 3a^2 & 3a & 1 \\ a^3 & a^2+2a & 2a+1 & 1 \\ a & 2a+1 & a+2 & 1 \\ 1 & 3 & 3 & 1 \end{bmatrix} = (a-1)^6$ 4

(c) Solve the equation $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$ 4

(d) Prove that if all the elements of one row (or one column) of a determinant are multiplied by the same number *k*, the value of the new determinant is *k* times the value of the given determinant. 4

UNIT–IV

7. (a) Define submatrix of a matrix, minors of a matrix and rank of a matrix. 1+1+2= 4

(b) Show that if *A* be an $m \times n$ matrix of rank *r*, then there exists non-singular matrices *P* and *Q* such that $PAQ = \begin{bmatrix} I_r & O \\ O & O \end{bmatrix}$ where I_r is a *r*-rowed unit matrix. 6

(c) Reduce the matrix $A = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ to the normal form $\begin{bmatrix} I_r & O \\ O & O \end{bmatrix}$ and hence find its rank. 4

8. (a) Show that the multiplication of the elements of a row by a non-zero number does not change the rank. 4

(b) Show that the elementary matrix corresponding to the E-operation $R_i \leftrightarrow R_j$ is its own inverse. 4

(c) Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$ to canonical (normal) form and hence find its rank. 6

UNIT–V

9. (a) Define sum of a subspace. Prove that

$$V = \left[\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \in R^3 / a_3 = a_1 + a_2 \right]$$ is a sub space of R^3. 1+3=4

(b) Find whether the following vectors are linearly independent or linearly dependent:

$V_1 = (1,2,3), V_2 = (1,0,0), V_3 = (0,1,0), V_4 = (0,0,1)$ 2

(c) Determine the characteristic roots of the matrix

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$$ 3

(d) State and prove Cayley-Hamilton theorem. 1+4=5

10. (a) Show that the set R^2 is a vector space. 5

(b) Prove that if $V_1, V_2, ..., V_n$ is a basis of $V(F)$ and if $w_1, w_2, ..., w_n \in V$ are linearly independent then $m \leq n$. 4

(c) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Hence, find A^{-1}. 5
