2023

B.A./B.Sc.

Second Semester

CORE – 3

STATISTICS

Course Code: STC 2.11

(Probability Distributions & Correlation Analysis)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Define random variable and its mathematical expectation. 1+2=3

   (b) What is the expectation of the number of failures preceding the first success in an infinite series of independent trials with constant probability p of success in each trail? 4

   (c) Write down the relationship between the moments and cumulants. 3

   (d) Let $f(x, y) = 8xy, 0 < x < y < 1$; $f(x, y) = 0$ elsewhere. Find

   (i) $E\left(\frac{Y}{X} = x\right)$ (ii) $E\left(\frac{XY}{X} = x\right)$

   (iii) $V\left(\frac{Y}{X} = x\right)$ 1+1+2=4

2. (a) Show that the mathematical expectation of the sum of two random variables is the sum of their individual expectations. 3

   (b) Given the following table:

x	–3	–2	–1	0	1	2	3
$P(x)$	0.05	0.10	0.30	0	0.30	0.15	0.10

   Compute:

   (i) $E(X)$ (ii) $E(4X+5)$

   (iii) $E(X^2)$ (iv) $V(X)$

   (v) $V(2X+3)$ 1+1½+1+1½+2=7

(c) Define the characteristic function of random variables. Prove that $\varnothing_X(t)$ is uniformly continuous in 't'. 1+3=4

UNIT–II

3. (a) Define Poisson distribution. Obtain its m.g.f. and prove the recurrence formula for the probabilities of Poisson distribution. 1+2+4=7

   (b) In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter 'p' of the distribution. 3

   (c) Find the mean and variance of the hyper-geometric distribution. 4

4. (a) Obtain the mean, variance, skewness, and kurtosis of binomial distribution. 7

   (b) Six coins are tossed 6400 times using the Poisson distribution; find the approximate probability of getting six heads r times. 3

   (c) Obtain binomial distribution as a limiting case of hyper– geometric distribution. 4

UNIT–III

5. (a) Prove that a linear combination of n independent normal variate is a normal variate. Obtain the m.g.f. of normal distribution. 5+2=7

   (b) If X is uniformly distributed with mean 1 and variance $\frac{4}{3}$, find $P(X<0)$. 3

   (c) Define the beta variate of second kind. Obtain its mean and variance. 1+3=4

6. (a) Define normal distribution. Discuss the chief characteristic of normal distribution. 1+6=7

   (b) Show that an exponential distribution "lacks memory", i.e. if X has an exponential distribution then for every constant $a \geq 0$, one has

   $P\left(Y \leq \frac{x}{X} \geq a\right) = p(X \leq x)$ for all x, where $Y = X - a$. 4

   (c) Define exponential distribution. Hence obtain its mean and variance. 1+1+1=3

UNIT–IV

7. (a) Show that the coefficient of correlation r is independent of change of scale and origin of the variable. Also prove that two independent variables are uncorrelated. 5+2=7

   (b) Define regression analysis. Prove that if one of the regression coefficients is greater than unity, the other must be less than unity. 1+2=3

   (c) Fit an exponential curve $Y = ab^X$ by the method of least square. 4

8. (a) The random variables X and Y are jointly normally distributed and U and V are defined by $U = X\cos\alpha + Y\sin\alpha$ and $V = Y\cos\alpha - X\sin\alpha$. Show that U and V will be uncorrelated if

$$\tan 2\alpha = \frac{2r\sigma_X\sigma_Y}{\sigma_X^2 - \sigma_Y^2}.$$ 4

   (b) Define the following: 2+1½+1½=5
   (i) Scatter diagram
   (ii) Tied ranks
   (iii) Repeated ranks

   (c) Let the line of regression of Y on X be $Y = a + bx$. Obtain the slope of the regression of Y on X, $b = \frac{\mu_{11}}{\sigma_X^2}$. 5

UNIT–V

9. (a) In the usual notations, prove that

$$R_{1.23}^2 = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{23}r_{31}}{1 - r_{23}^2} \le r_{12}^2.$$ 6

   (b) Find if A and B are independent, positively associated or negatively associated, in each of the following cases: 1+1½+1½=4
   (i) $N = 1000$, $(A) = 470$, $(B) = 620$ and $(AB) = 320$
   (ii) $(A) = 490$, $(AB) = 294$, $(\alpha) = 570$, and $(\alpha B) = 380$
   (iii) $(AB) = 256$, $(\alpha B) = 768$, $(A\beta) = 48$, and $(\alpha\beta) = 144$

   (c) Prove multiple correlations in terms of total and partial correlations. 4

10. (a) If r_{12} and r_{13} are given, show that r_{23} must lie in the range $r_{12}r_{13} \pm (1-r_{12}^2-r_{13}^2+r_{12}^2r_{13}^2)^{\frac{1}{2}}$. Also if $r_{12}=k$ and $r_{13}=-k$, prove that r_{23} will lie between -1 and $1-2k^2$. 5+4=9

(b) If $1-R_{1.23}^2=(1-r_{12}^2)(1-r_{13.2}^2)$, then prove

(i) $R_{1.23} \geq r_{12}$

(ii) X_1 is uncorrelated with any of the variable when $R_{1.23}=0$

2+3=5
