

2023
B.A./B.Sc.
Sixth Semester
DISCIPLINE SPECIFIC ELECTIVE – 3
PHYSICS
Course Code: PHD 6.11
(Advanced Mathematical Physics - II)

Total Mark: 70
Time: 3 hours

Pass Mark: 28

Answer five questions, taking one from each unit.

UNIT-I

1. (a) Two particles of masses m_1 and m_2 move under the action of their gravitational interaction, find the Lagrangian equation of motion of the particles. 4
- (b) Define geodesic. Show that the shortest distance between two points in a plane is a straight line. 1+3=4
- (c) Explain the principle of virtual work and derive the D'Alembert's principle from the principle of virtual work. 6
2. (a) Explain the terms degrees of freedom and generalized coordinates with examples. 4
- (b) Obtain the equation of motion of a system of two masses, connected by an inextensible string passing over a small smooth pulley. 4
- (c) A particle slides from rest at one point on a frictionless wire in a vertical plane to another point under the influence of the earth's gravitational field. If the particle travels in the shortest time, show that the path followed by it is a cycloid. 6

UNIT-II

3. (a) Show that the transformation $Q = \log \left(\frac{1}{q} \sin p \right)$; $P = qc \cot p$ is canonical. 4

- (b) Use Hamilton's principle to find the equation of motion of one-dimensional harmonic oscillator. 4
- (c) Prove that for any three functions F , G and K of p_k and q_k , the following relation holds true:
 $[F, [G, K]] + [G, [K, F]] + [K, [F, G]] = 0$ 6
4. (a) Obtain the necessary relations for Legendre transformations. 4
 (b) Write the Hamiltonian for a simple pendulum and deduce its equation of motion. 4
 (c) Prove that $[J_z, J_y] = -J_x$ 6

UNIT-III

5. (a) In Z , we define $a * b = a + b + 1$. Show that $(Z, *)$ is an abelian group. 6
 (b) Verify the group property of symmetry of equilateral triangle (D_3) also known as C_{3v} . 8
6. (a) Show that the cube root of unity is an abelian group under multiplication. 6
 (b) If $a, b \in G$ then show that the equation $a * x = b$ and $y * a = b$ have unique solution in the group G . 8

UNIT-IV

7. (a) There are two urns. The 1st urn contains three bags containing 5 white and 3 black balls. The 2nd urn contains 2 bags containing 2 white and 4 black balls. A white ball is drawn at random. What is the probability that this ball has been drawn from the 1st urn? 4
 (b) If A and B be any two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$, and

$$P(A \cup B) = \frac{1}{2}. \text{ Find}$$

(i) $P(B/A)$

(ii) $P(A \cup B')$

(iii) $P(A/B')$ 4

- (c) A continuous random variable X takes values in the interval $1 \leq x \leq 3$. The probability density function of X is given by

$$f(x) = \frac{k}{x^2}.$$

- (i) Determine the value of K .
 (ii) Find $E(x)$ and variance of X .

6

8. (a) Determine the co-efficient of $x^{11}y^4$ in the expansion of $(2x^2 - 3xy^2 + z^2)^6$.
 (b) The probability that a pen manufactured by a company will be defective is $2/10$. If 10 such pens are manufactured, find the probability that
 (i) at least 2 will be defective
 (ii) exactly 3 will be defective
 (c) A random variable X has the following probability distribution

4

X	0	1	2	3	4	5	6	7
$P(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2 + K$

5

Find $P\left(\frac{1.5 < x < 4.5}{x > 2}\right)$

5

UNIT-V

9. (a) A manufacturer knows that the condenser he makes contain on an average 1% defective. He packs them in boxes of 100. What is the probability that a box picked at random will contain 3 or more defective condenser?
 (b) Find the first four moments of binomial distribution.
 10. (a) Using the method of least square, fit a straight line to the four points.

4

10

x	1	2	3	4
y	1.7	1.8	2.3	3.2

4

(b) The distribution of 500 workers in a factory is approximately normal with mean Rs. 75 and standard deviation Rs.15. Find the number of workers who receive weekly wages

(i) More than Rs. 90

(ii) Less than Rs. 46

5

(c) The following data are the number of seeds germinating out of 10 on damp filter for 80 seeds. Fit a binomial distribution to these data:

x	0	1	2	3	4	5	6	7	8	9	10
f	6	20	23	12	8	6	0	0	0	0	0

5
