2023

B.A./B.Sc.

Fourth Semester

CORE – 8

PHYSICS

Course Code: PHC 4.11

(Mathematical Physics - III)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Evaluate $\left(\dfrac{1+\sin\alpha+i\cos\alpha}{1+\sin\alpha-i\cos\alpha}\right)^n$. 4

   (b) Find the root of the complex number $x^4+i=0$. 5

   (c) If $\alpha, \alpha^2, \alpha^3, \alpha^4$ are the roots of $x^5-1=0$ find them and show that $(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4)=5$. 5

2. (a) Prove the necessary condition for the function $f(z)$ to be analytic. 5

   (b) Show that the function $e^x(\cos y+i\sin y)$ is analytic function, real and imaginary. Find its derivative. 5

   (c) Determine whether $\dfrac{1}{z}$ is analytic or not. 4

UNIT–II

3. (a) Evaluate $\int_{1-i}^{2+i}(2x+iy+1)dz$ along the straight line joining 1–*i* and 2+*i*. 4

   (b) Expand the following function in Taylor series $f(z)=\dfrac{z-1}{z+1}$ about $z = 0$. 5

   (c) Evaluate $\oint \tan z.dz$, where C is circle $|z|=2$. 5

4. (a) Determine the poles and residue of the function $f(z)=\dfrac{z}{z-1}$. 4

(b) Using residue theorem, evaluate $\int \dfrac{12z-7}{(z-1)^2(2z+3)}$, Where C is the circle $|z|=2$. 5

(c) Evaluate $\int_0^{2\pi} \dfrac{d\theta}{5-3\cos\theta}$ 5

UNIT–III

5. (a) Express the function $f(x)=\begin{cases}1, & \text{when } |x|\le 1\\ 0, & \text{when } |x|>1\end{cases}$ as a Fourier integral.

Hence evaluate $\int_0^{\infty} \dfrac{\sin\lambda\cos\lambda x}{\lambda} d\lambda$ 5

(b) Find the complex form of Fourier integral representation of

$$f(x)=\begin{cases}e^{-kx}, & x>0 \text{ and } k>0\\ 0, & \text{otherwise}\end{cases}$$ 5

(c) Find the Fourier transform of e^{-ax^2} ,where $a>0$. 4

6. (a) Obtain Fourier cosine transform of

$$f(x)=\begin{cases}x, & \text{for } 0<x<1\\ 2-x, & \text{for } 1<x<2\\ 0, & \text{for } x>2\end{cases}$$ 5

(b) State and prove the convolution theorem on Fourier transform. 5

(c) Show that $F\{f(x)\cos ax\}=\dfrac{1}{2}[F(s+a)+F(s-a)]$ 4

UNIT–IV

7. (a) Find the Laplace transforms of

(i) $f(x)=\begin{cases}t-1, & 1<t<2\\ 3-t, & 2<t<3\end{cases}$ 5

(ii) $f(x) = t^3$, using 3rd order derivatives. 4

(b) Solve the initial value problem $y'' + 4y = 0, y(0) = 1, y'(0) = 6$ 5

8. (a) Find $L^{-1}\left[\frac{1}{s^2(s^2+4)}\right]$ using Laplace transform of integral. 6

(b) Find the inverse Laplace transform (first shifting theorem) of

$$\frac{6+s}{s^2+6s+13}$$ 4

(c) Find the Laplace transform of the waveform

$$f(t) = \left(\frac{2t}{3}\right), 0 \le t \le 3$$ 4

UNIT–V

9. (a) Solve $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ for $x \ge 0, t \ge 0$ under the given conditions $u = u_0$ at $x = 0, t > 0$ with initial condition $u(x,0) = 0, x \ge 0$. 7

(b) Use Fourier sine transform to solve the equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ under the conditions

(i) $u(0,t) = 0$

(ii) $u(x,0) = e^{-x}$

(iii) $u(x,t)$ is bounded 7

10. (a) A resistance *R* in series with inductance (*L*) is connected with e.m.f. *E*(*t*). The current *i* is given by $L\frac{di}{dt} + Ri = E(t)$ if the switch is connected at $t = 0$ and disconnected at $t = a$. Find the current *i* in terms of *t*. 8

(b) Solve $\frac{dx}{dt} + y = 0$ and $\frac{dx}{dt} - x = 0$ under the condition $x(0) = 1, y(0) = 0$. 6