2023

B.A./B.Sc.

Fourth Semester

GENERIC ELECTIVE – 4

MATHEMATICS

Course Code: MAG 4.11

(Differential Equations & Higher Trigonometry)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Reduce to homogeneous form and solve

$(2x+2y+3)dy=(x+y+1)dx$ 5

(b) Solve:

(i) $(x^2D^2-3xD+5)y=\sin(\log x)$ 5

(ii) $x^2\dfrac{d^2y}{dx^2}+y=3x^2$ 4

2. (a) Solve $\dfrac{d^2y}{dx^2}+4y=0$, given that $y=2$ and $\dfrac{dy}{dx}=0$ when $x=0$. 4

(b) Solve:

(i) $(D^2-1)y=e^x\cos x$ 5

(ii) $(D^2-4D+4)y=e^{-4x}+5\cos 3x$ 5

UNIT–II

3. (a) Solve the Lagrange's equation $y=2px-p^2$, where $p=\dfrac{dy}{dx}$. 4

(b) Find the integrating factor and solve the equation

$(y^4+2y)dx+(xy^3+2y^4-4x)dy=0$ 4

(c) Find the general and singular solution of $3xy = 2px^2 - 2p^2$, where $p = \frac{dy}{dx}$. 6

4. (a) Find the integrating factor and solve the equation $y(x^2y^2+2)dx + x(2-2x^2y^2)dy = 0$ 4

(b) If $p = \frac{dy}{dx}$, solve $p(p+x) = y(x+y)$. 5

(c) Find the solution of the differential equation

$9p^2(2-y)^2 = 4(3-y)$ where $p = \frac{dy}{dx}$.

Also, find the singular solution. 5

UNIT–III

5. (a) Solve $x^2\frac{d^2y}{dx^2} - (x^2+2x)\frac{dy}{dx} + (x+2)y = x^3e^x$ 5

(b) Solve the ordinary simultaneous equations

$$\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2\cos t - 7\sin t$$

$$\frac{dx}{dt} - \frac{dy}{dt} + 2x = 4\cos t - 3\sin t$$ 4

(c) Solve $(2xz - yz)dx + (2yz - zx)dy - (x^2 - xy + y^2)dz = 0$ 5

6. (a) Solve $\frac{d^2y}{dx^2} - \frac{2}{x}\frac{dy}{dx} + (1 + \frac{2}{x^2})y = xe^x$ by changing the dependent variable. 5

(b) Solve $\frac{yzdx}{y-z} = \frac{zxdy}{z-x} = \frac{xydz}{x-y}$ 4

(c) Verify for integrability and solve $(y^2+yz)dx + (xz+z^2)dy + (y^2-xy)dz = 0$ 5

UNIT–IV

7. (a) Using De Moivre's theorem solve $x^9 - x^5 + x^4 - 1 = 0$. 4

(b) If $\theta_1, \theta_2, \theta_3$ are the values of θ which satisfy the equation $\tan 2\theta = \lambda \tan(\theta + \alpha)$, and if no two of these values differ by a multiple of π, show that $(\theta_1 + \theta_2 + \theta_3 + \alpha)$ is a multiple of π. 4

(c) Prove that the roots of the equation $x^3 - 3\sqrt{3}x^2 - 3x + \sqrt{3} = 0$ are $\tan\frac{\pi}{9}, \tan\frac{4\pi}{9}, \tan\frac{7\pi}{9}$. 6

8. (a) Simplify $\dfrac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5}$ 4

(b) Expand $\cos^9\theta$ in a series of cosines of multiples of θ. 5

(c) Prove that $64(\cos^8\theta + \sin^8\theta) = \cos 8\theta + 28\cos 4\theta + 35$. 5

UNIT–V

9. (a) If $\tan(\theta + i\phi) = \sin(x + iy)$, then prove that $\coth y \sinh 2\phi = \cot x \sin 2\theta$. 4

(b) Show that $i\log\dfrac{x-i}{x+i} = \pi - 2\tan^{-1}x$. 4

(c) Show that

$$\cos^{-1}(\cos\theta + i\sin\theta) = \sin^{-1}\sqrt{\sin\theta} + i\log\left\{\sqrt{1+\sin\theta} - \sqrt{\sin\theta}\right\}$$

where θ is a positive acute angle. 6

10. (a) Separate $Log\sin(x + iy)$ into real and imaginary parts. 4

(b) Prove that

(i) $\sin^{-1}(ix) = -i\log\left(\sqrt{x^2+1} - x\right) = i\log\left(\sqrt{x^2+1} + x\right)$ 5

(ii) $\dfrac{\pi}{4} = \dfrac{17}{21} - \dfrac{713}{81\times 343} + \ldots + \dfrac{(-1)^{n+1}}{2n-1}\left\{\dfrac{2}{3}9^{1-n} + 7^{1-2n}\right\} + \ldots$ 5
