2023

B.A./B.Sc.

Sixth Semester

DISCIPLINE SPECIFIC ELECTIVE – 3

MATHEMATICS

Course Code: MAD 6.11

(Theory of Equations)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Without actual division, show that $(x-1)^{2n} - x^{2n} + 2x - 1$ is divisible by $2x^3 - 3x^2 + x$. 4

(b) In the polynomial $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n, a_n \neq 0$, prove that the leading term a_0x^n exceeds the sum of the remaining terms for $x \leq \frac{a_k}{a_0} + 1$, where a_k is the greatest coefficient irrespective of signs. 4

(c) Find a relation between a and b 3×2 = 6

(i) If $4x^3 - 3x^2 + 2ax + b$ is exactly divisible by $(x+2)$

(i) If $ax^5 + 3bx^3 + 8$ is exactly divisible by $(x-2)$

2. (a) Prove that every equation of degree n has exactly n roots. 5

(b) Find the condition that $x^3 + px^2 + qx + r = 0$, may have the sum of its two root equal to zero. 4

(c) Find the condition that the roots of the equation $x^4 + px^2 + qx + r = 0$ are in AP. 5

UNIT–II

3. (a) Transform the equation to an equation with integral coefficients and its leading coefficient equals to unity: 3×2=6

(i) $x^4-\frac{1}{3}x^3+\frac{2}{9}x^2-\frac{4}{27}x-\frac{2}{81}=0$

(ii) $4x^4+3x^3-4x^2-5x+2=0$

(b) Find the equations whose roots are the cubes of the roots of the equation $x^4-2x^3+x^2+3x-1=0$. 4

(c) Remove the second term of the equation $x^3+6x^2+9x+4=0$ and hence solve it. 4

4. (a) Deduce an equation whose roots are the squared difference of the roots of the $x^3+x^2-x=1$, and hence show that two roots of this equations are equal. 5

(b) Show that the equation $x^4-3x^3+4x^2-2x+1=0$ can be transformed into a reciprocal equation by diminishing the roots by unity. Hence solve it. 5

(c) Prove that the special roots of the equation $x^9-1=0$ are the roots of the equation $x^6+x^3+1=0$ and their values are

$\cos\frac{2r\pi}{9}\pm i\sin\frac{2r\pi}{9}, r=1,2,4$. 4

UNIT–III

5. (a) Solve $x^3+3x^2+6x+4=0$ by Cardan's method. 5

(b) Solve $x^3-3x^2+3=0$. 4

(c) Show that the equation $x^4-9x^2+4x+12=0$ has equal roots and hence solve it. 5

6. (a) Find the Euler's cubic of the equation $x^4-3x^3+5x^2-5x+2=0$ and hence solve it using Euler's method. 7

(b) Solve the equation $x^4-18x^2+32x-15=0$ by Ferrari's method. 7

UNIT–IV

7. (a) If α, β, γ be the roots of the equation $x^3 + 2x^2 + 3x + 4 = 0$, then find the equation whose roots are 3×3=9

   (i) $1+\frac{1}{\alpha}, 1+\frac{1}{\beta}, 1+\frac{1}{\gamma}$

   (ii) $\alpha - \frac{1}{\beta\gamma}, \beta - \frac{1}{\gamma\alpha}, \gamma - \frac{1}{\alpha\beta}$

   (iii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}, \frac{\beta}{\gamma}+\frac{\gamma}{\beta}, \frac{\gamma}{\alpha}+\frac{\alpha}{\gamma}$

   (b) If $\alpha, \beta, \gamma, \delta$ be the roots of the equation $x^4 + px^2 + qx + r = 0$, then show that $\sum \alpha^6 = 6pr + 3q^2 - 2p^3$ and $\sum \alpha^7 = -7q(p^2 - r)$. 5

8. (a) If α, β, γ be the roots of the equation $x^3 + qx + r = 0$, then show that 3×3=9

   (i) $\alpha^3 + \beta^3 + \gamma^3 = -3r$

   (ii) $\alpha^5 + \beta^5 + \gamma^5 + 5\alpha\beta\gamma(\beta\gamma + \gamma\alpha + \alpha\beta) = 0$

   (iii) $\sum \frac{1}{\alpha^2 - \beta\gamma} = -\frac{3}{q}$

   (b) Find the limits of the roots of the equation $x^5 + x^4 + x^2 - 25x - 100 = 0$. 5

UNIT–V

9. (a) Analyse completely the equation $x^5 + x^4 + x^2 - 25x - 36 = 0$. 5

   (b) Find the conditions that the roots of the biquadratic $z^4 + 6Hz^2 + 4Gz + a^2I - 3H^3 = 0$ are all real. 4

   (c) Use Sturm's theorem to prove that the equation $x^3 - 7x + 7 = 0$, has two roots between 1 and 2 and one root between (–4) and (–3). 5

10. (a) Find the integral root of the equation $x^4 + x^3 - 2x^2 + 4x - 24 = 0$. 3

(b) Using Newton's method of approximation, find the positive root of the equation $x^3 - 2x - 5 = 0$, correct to three decimal places. 5

(c) Find by Horner's method, the real positive root of $8x^3 - 10x^2 - 3x - 7 = 0$ which lies between 1 and 2, correct to four places of decimal. 6