2023
B.A./B.Sc.
Fourth Semester
CORE – 8
MATHEMATICS
Course Code: MAC 4.11
(Numerical Methods)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Define absolute error and relative error. 2
(b) Obtain a second degree polynomial approximation to $f(x)=(1+x)^{\frac{1}{2}}$, $x \in [0, 0.1]$ using Taylor series expansion about $x=0$. Use this expansion to approximate $f(0.05)$. Also, find the bound of the truncation error. 5
(c) Design an algorithm to generate the Fibonacci sequence with n terms and draw the flow chart. 7

2. (a) Three approximate values of the number $\frac{1}{3}$ are 0.30, 0.33 and 0.34. Which of these three values is of the best approximation? 2
(b) List five important characteristics of a good algorithm. 5
(c) Design an algorithm to find the sum of the series $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}$, where n is a positive integer. 7

UNIT–II

3. (a) Given the equation $x^4-x-10=0$, determine the initial approximation to find the smallest positive root. Use regula-falsi method to find the root correct to three decimal places. 7

(b) Define rate of convergence. Show that the rate of convergence for bisection method is linear. 7

4. (a) Perform five iterations on the bisection method to obtain the root of the equation $\cos x - xe^{x} = 0$ in the interval [0, 1]. 7

(b) Perform five iterations of the Newton's method to obtain the approximate value of $17^{1/3}$ starting with the initial approximation $x_0 = 2$. 7

UNIT–III

5. (a) Solve the system of linear equation given in matrix form using Gaussian elimination method with partial pivoting 7

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$

(b) Prove that the necessary and sufficient condition for convergence of an iterative method of the form $X^{(k+1)} = HX^{(k)} + c$ is that the eigen values of the iteration matrix satisfies $|\lambda_i| < 1$. 7

6. (a) Find the necessary and sufficient condition of *k* for Gauss-Seidel method to converge for a system of equation *AX*=*B*, where

$$A = \begin{bmatrix} 1 & 0 & k \\ 2 & 1 & 3 \\ k & 0 & 1 \end{bmatrix}$$ and *B* is arbitrary. Also, find the eigen values of the iteration matrix for $k = \frac{1}{2}$ and rate of convergence of the method.

4+3=7

(b) Solve the system of equations

$$4x_1 + 2x_2 + x_3 = 4$$
$$x_1 + 3x_2 + x_3 = 4$$
$$3x_1 + 2x_2 + 6x_3 = 7$$

using Gauss-Jacobi method in error format. Perform three iterations using the initial approximation $X^{(0)} = \begin{bmatrix}0.1 & 0.8 & 0.5\end{bmatrix}^T$. 7

UNIT–IV

7. (a) Let $f(x) = \ln(1+x), x_0 = 1$ and $x_1 = 1.1$. Use linear interpolation to calculate an approximate value of $f(1.04)$ and obtain a bound on the truncation error. 5

(b) Evaluate $\Delta^3(1-x)(1-2x)(1-3x)$. 2

(c) The population of a country in the decennial census were under.

Year x:	1941	1951	1961	1971	1981
Population (in Lakhs) y:	46	67	83	95	102

Calculate the differences and obtain the polynomial using Gregory-Newton backward interpolation. Estimate the population for the year 1975. 7

8. (a) Prove the relation : 5

(i) $\mu = \sqrt{\frac{\delta^2}{4} + 1}$

(ii) $\nabla = 1 - (1+\Delta)^{-1}$

(b) Prove that the n^{th} divided difference of a polynomial x^n is 1. 5

(c) Using Lagrange's interpolation formula, prove that

$y_1 = y_3 - 0.3(y_5 - y_{-3}) + 0.2(y_{-3} - y_{-5})$. 4

UNIT–V

9. (a) Calculate by Boole's rule an approximate value of $\int_3^{-3} x^4 dx$, taking five ordinates. 4

(b) Use Simpson's 3/8th rule to obtain the value of $\int_0^1 \frac{dx}{1+x^2}$. 5

(c) Using Euler's method, find the approximate value of y at $x = 0.1$ in five steps, given $\frac{dy}{dx} = x + y$ and $y(0) = 1$. 5

10. (a) A rocket is launched from the ground at time $t = 0$. Its acceleration a is registered during the first 80 seconds and is given in the table below:

t (sec)	0	10	20	30	40	50	60	70	80
a (m/sec^2)	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

Find the velocity of the rocket at time t = 80 seconds. 7

(b) Use Runge-Kutta fourth order method to solve $\frac{dy}{dx} = xy$ for $x = 1.4$. Initially $x = 1, y = 2$. (Take stepsize $h = 0.2$) 7