

**April 2025**  
**B.A./B.Sc.**  
**Second Semester**  
**MINOR – 2**  
**STATISTICS**  
*Course Code: STN 2.11*  
(Probability Distribution & Correlation Analysis)

Total Mark: 50

Pass Mark: 20

Time: 2 hours

I. Answer any three questions, taking one from each unit.

**UNIT-I**

1. (a)  $\text{var}(X + C) = \underline{\hspace{2cm}}$ , where  $C$  is a constant. Prove that  
 $\text{cov}(ax + b, cY + d) = ac \text{cov}(X, Y)$ . 1+2=3
- (b) A coin is tossed until a head appears. What is the expectation of the number of tosses required? 4
- (c) Prove Chebyshev's inequality. 5
2. (a) Prove the variance of linear combination of random variables. 4
- (b) If  $X$  and  $Y$  are two independent random variables, show that  
 $\text{var}(aX + bY) = a^2V(X) + b^2V(Y)$ . 3
- (c) Two random variables  $X$  and  $Y$  have the following joint probability density function. 5

$$f(x, y) = \begin{cases} k(4 - x - y); & 0 \leq x \leq 2; 0 \leq y \leq 2 \\ 0; & \text{otherwise} \end{cases}$$

Find:

- (i) the constant  $k$
- (ii) marginal density functions of  $X$  and  $Y$
- (iii) conditional density functions and
- (iv)  $\text{var}(X)$ ,  $\text{var}(Y)$  and  $\text{cov}(X, Y)$

**UNIT-II**

3. (a) Find the mean and variance of Bernoulli distribution. Define the binomial distribution with the parameters  $p$  and  $n$ . Obtain the

m.g.f. of binomial distribution and hence find its mean, variance, skewness, and kurtosis. 1+1+7=9

(b) Show the moments of rectangular distribution. 3

4. (a) Derive Poisson distribution as a limiting form of a binomial distribution. Find the m.g.f. of exponential distribution. 5+3=8

(b) A coffee connoisseur claims that he can distinguish between a cup of instant coffee and a cup of percolator coffee 75% of the time. It is agreed that his claims will be accepted if he correctly identifies at least 5 of the 6 cups. Find his chances of having the claim

(i) accepted

(ii) rejected

when he does have the ability he claims. 4

### UNIT-III

5. (a) Prove that Spearman's rank correlation coefficient is given by

$1 - \frac{6 \sum d_i^2}{n^2 - n}$ , where  $d_i$  denotes the difference between the ranks of  $i^{\text{th}}$  individual. 6

(b) Why are there two lines of regression? Fit an exponential curve by the method of least square. 6

6. (a) Prove that two independent variables are uncorrelated. 2

(b) Show that the partial correlation coefficient between  $X_1$  and

$X_2$  is  $r_{12.3} = \frac{\text{cov}(X_{1.3}, X_{2.3})}{\sqrt{\text{var}(X_{1.3}) \text{var}(X_{2.3})}}$ . 6

(c) Explain the two properties of residuals. 4

II. Answer any two of the following questions.

7. (a) A coin is tossed four times. Let  $X$  denote the number of times a head appears. Find the distribution, mean and variance of  $X$ . 4

(b) Write down the relationship between moments and cumulants. 3

8. Define normal distribution. Mention any five chief characteristics of normal distribution and obtain its m.g.f. 2+3+2=7

9. (a) The random variables  $X$  and  $Y$  are jointly normally distributed, and  $U$  and  $V$  are defined by  $U = X \cos \alpha + Y \sin \alpha$  and  $V = Y \cos \alpha - X \sin \alpha$ . Show that  $U$  and  $V$  will be uncorrelated if
- $$\tan 2\alpha = \frac{2r\sigma_X\sigma_Y}{\sigma_X^2 - \sigma_Y^2}. \quad 4$$

- (b) Fit a second degree equation  $Y = a + bx + cx^2$  by the method of least square. 3
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