April 2025
B.A./B.Sc.
Second Semester
MAJOR – 2
STATISTICS
Course Code: STM 2.11
(Probability Distributions & Correlation Analysis)

Total Mark: 50 *Pass Mark: 20*
Time: 2 hours

I. Answer three *questions, taking* one *from each unit.*

UNIT-I

1. (a) Two random variables *x* and *y* have the following joint probability density function.

$$f(x,y)=\begin{cases} 2-x-y; & 0\le x\le 1, 0\le y\le 1 \\ 0; & \text{otherwise} \end{cases}$$

Find:
(i) Marginal probability density function of *x* and *y*.
(ii) Conditional density functions
(iii) Var(*x*) and Var(y). 2+2+2=6

(b) Define cumulants and obtain the first four cumulants in terms of central moments. 6

2. (a) Define moments and moment generating function of a random variable *X*. If *M*(*t*) is the MGF of a random variable *X* about the origin, show that the moment $\mu_r^{'}$ is given by

$$\mu_r^{'}=\left[\frac{d^r\mu(t)}{dt^r}\right]t=0.$$ 1+2+3=6

(b) State and prove weak law of large numbers (WLLN). 6

UNIT-II

3. (a) Define uniform distribution with usual notations. Find mean, variance, and moment generating function of uniform distribution. 6

(b) Obtain the moment generating function of binomial distribution and hence find the value of mean and variance of binomial distribution. 3+3=6

4. (a) Define Poisson distribution. Obtain its mean, variance and moment generating function. 1+1+2+2=6

(b) Write four properties of normal distribution. Prove that a linear combination of independent normal variate is also a normal variate. 2+4=6

UNIT-III

5. (a) Explain scatter diagram. 3

(b) The random variables X and Y are jointly normally distributed, and U and V are defined by $U = X\cos\alpha + Y\sin\alpha$ and $V = Y\cos\alpha - X\sin\alpha$. Show that U and V will be uncorrelated if $\tan 2\alpha = \dfrac{2r\sigma_X\sigma_Y}{\sigma_X^2\sigma_Y^2}$. 4

(c) State and prove any two properties of regression. 5

6. (a) Show that the coefficient of correlation is independent of change of origin and scale. 5

(b) Define multiple correlation. Write the properties of multiple correlation coefficient. 1+2=3

(c) Obtain the normal equations for the second-degree polynomial using principle of least square method. 4

II. Answer any two *of the following questions.*

7. (a) Let X be a random variable with the following probability distribution:

x	–3	6	9
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find $E(X)$ and $E(X^2)$ using the law of expectation, evaluate $E(2X+1)^2$. 5

(b) State the Chebyshev's inequality. 2

8. (a) Obtain the recurrence relation for the probabilities of binomial distribution. 3

(b) Find the normal distribution as a limiting form of binomial distribution. 4

9. (a) Define rank correlation. 1

(b) Show that:

$$1 - R_{1.23}^2 = (1 - r_{12}^2)(1 - r_{13.2}^2)$$

using the above result prove that:

$$1 - R_{1.23}^2 = \frac{(1-\rho)(1+2\rho)}{(1+\rho)}$$

provided all coefficient of zero order are equal to ρ. 3+3=6