April 2025
B.A./B.Sc.
Fourth Semester
CORE – 9
STATISTICS
Course Code: STC 4.21
(Linear Models)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) If $I'\beta$ is any estimable linear function of the parameters $\beta_1,\beta_2,\ldots,\beta_P$ then: 4+3=7
   (i) $\exists$ a unique linear function $C'Y$ of the random variables $Y_1,Y_2,\ldots Y_n$ such that $c \in V(A')$ and $E(c'Y) = I'\beta C'Y$
   (ii) $V(C'Y)$ is less than the variance of any other linear unbiased estimator of $I'\beta$
   (b) Define the following: 2×2=4
   (i) Least-square estimators
   (ii) Normal equations
   (c) Show that the best estimator of the estimable linear function $I'\beta$ of the parameters is $I'\hat{\beta}$, where $\hat{\beta}_1,\hat{\beta}_2,\cdots\hat{\beta}_P$ are a set of LS estimators of $\beta_1,\beta_2,\cdots\beta_P$.
   3

2. (a) Prove the Gauss-Markov linear model in terms of matrix notations. When is the model called an analysis of variance model, regression model and analysis of covariance model? 4+3=7
   (b) What is a BLUE? Define parametric function and estimable function. 3+2+2=7

UNIT–II

3. (a) Describe the test for the linearity of regression. 7
   (b) Delineate the test for equality of regression equation from p group using the analysis of variance technique. 7

4. (a) Describe the test for the homogeneity of a group of regression coefficients. 7
   (b) Show the test for multiple linear regression model. 7

UNIT–III

5. (a) Explain the meaning of "analysis of variance" and give its uses. In ANOVA testing for one-way classification (for fixed effect model) if the null hypothesis $H_0 : \mu_1 = \mu_2 = \cdots\ldots = \mu_k$ is rejected, how will you proceed to test the significance of the difference between any two treatment means? 4+5=9
   (b) For a two-way ANOVA obtain (estimate): 5
   (i) The linear model
   (ii) The parameters

6. (a) For a two-way ANOVA give the layout. Differentiate between fixed effect model and random effect model. Also, give all the assumptions made in ANOVA. 3+1+3=7
   (b) Give the statistical analysis (fixed effect model) for one way ANOVA. Explain the hypothesis to be used in two-way ANOVA. Also, give the ANOVA table for two-way classification. 3+4=7

UNIT–IV

7. (a) Describe the "analysis of covariance for a RBD with one concomitant only". Explain analysis of covariance with some examples. 4+3=7
   (b) Give the variance ratio (F) value for one-way ANCOVA. For one-way ANCOVA linear model, $y_{ij} = \mu_i + \beta\left(x_{ij} - \bar{x}_{..}\right) + e_{ij}$ use the principle of least squares and estimate all the parameters, hence prove that $E_{YY} = T_{YY} - B_{YY}$. 1+6=7

8. (a) Derive the formula for total sum of product between variable X and Y. Give the null hypothesis for RBD and the ANCOVA table for RBD. 3+4=7

   (b) Derive the relation for restricted residual sum of squares (or error S.S) for Model-1. Differentiate between one-way and two-way ANCOVA. 5+2=7

UNIT–V

9. (a) Define econometrics and explain its two types. Show the various steps involved in the methodology of econometrics. 4+3=7

   (b) Prove that the standard errors of the estimates become infinitely large. 3

   (c) Elucidate the marginal propensity to consume. Peter's income in the month of July was ₹ 8,000 and his consumption for the month of July was ₹ 5,000. But in the month of August his income increases to ₹ 10,000 and his consumption also increases to ₹ 6,000. How much percent of the increased income did he use for the purpose of consumption? 2+2=4

10. (a) How can econometrics be used as a tool for forecasting and prediction? 2

    (b) Explain the concept of multicollinearity. 5

    (c) What do you mean by autocorrelation? Delineate the detection of autocorrelation. 2+5=7