April 2025
B.A./B.Sc.
Fourth Semester
CORE – 8
STATISTICS
Course Code: STC 4.11
(Statistical Inference)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Define estimator with example. Show that in a random sampling from a normal population, sample mean is a consistent estimator of population mean. 2+4=6
   (b) What is efficiency? If T is the most efficient estimator and T' is any other unbiased estimator with efficiency 'e', find the efficiency of $\frac{1}{2}(T+T')$. 1+4=5
   (c) Derive Fisher-Neyman factorisation theorem. 3

2. (a) Mention all the properties of a good estimator. Also, define a good estimator. 2+1=3
   (b) Show that sample mean is an unbiased estimator of population mean. 4
   (c) If $X_1, X_2, \ldots, X_n$ be a random sample drawn from binomial distribution with parameters n and p. Then, show that $\frac{\theta(\theta-1)}{n(n-1)}$ in an unbiased estimator of p^2 where $\theta = \sum_{n=1}^{n} X_i$. 5
   (d) Define most efficient estimator and sufficient estimator. 2

UNIT–II

3. (a) Define Cramer-Rao inequality. Give all the regularity conditions for Cramer-Rao inequality. 2+3=5

(b) Obtain the MVB estimator for μ in normal population $N(\mu, \sigma^2)$, where σ^2 is known. 4

(c) Define confidence interval and confidence limits. A random sample of 15 observation has mean 20 and standard deviation 3.5. Determine the

(i) margin of error

(ii) confidence interval at 95% level of significance.

$(t_{0.25,14} = 2.145)$ 2+3=5

4. (a) Derive the equality sign in Cramer-Rao inequality. Thus, define minimum variance bound (MVB). 4

(b) State and prove Rao-Blackwell theorem. 6

(c) Derive the theorem for confidence limits and confidence interval at 1% level of significance. 4

UNIT–III

5. (a) Let $x_1, x_2, ..., x_n$ be a random sample from the uniform distribution with p.d.f. $f(x, \theta) = \begin{cases} \frac{1}{\theta}, 0 < x < \infty, \theta > 0 \\ 0, \quad \text{elsewhere} \end{cases}$

Obtain the maximum likelihood estimator for θ. 5

(b) How can one estimate the parameters of a distribution by the method of minimum chi-square? 4

(c) Estimate α and β in the case of Pearson's type III distribution by the method of moments:

$$f(x; \alpha, \beta) = \frac{\beta^\alpha}{y(\alpha)} x^{\alpha-1} e^{-\beta x}, 0 \leq x < \infty$$ 5

6. (a) Prove that the M.L.E. of the parameter α of a population having density function

$$\frac{2}{\alpha^2}(\alpha - x); 0 < x < \alpha$$

for a sample of unit is $2x$, x being the sample value. Also, show that the estimate is biased. 5

(b) Describe the method of moments for estimating the parameters. What are the properties of the estimates obtained by this method? 3+2=5

(c) Find Bayes estimator of the parameter p of a binomial distribution with x successes out of n given that the prior distribution of p is a beta distribution with parameters α and β. 4

UNIT–IV

7. (a) Define most powerful test. 2

(b) Every most powerful or uniformly most powerful critical region is necessarily unbiased. Prove that if W be an MPCR of size α for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, then it is necessarily unbiased. 6

(c) Describe likelihood ratio test and state its important properties. 4+2=6

8. (a) Explain optimum region and sufficient statistics. 4

(b) Given the frequency function:

$$f(x, \theta) = \begin{cases} \frac{1}{\theta}, 0 \le x \le 0 \\ 0, \text{elsewhere} \end{cases}$$

and that the null hypothesis to be tested is $H_0 : \theta = 1$ against $H_0 : \theta = 2$, by means of single observed value of x. What would be the sizes of type I and type II errors, if the interval is $0.5 \le x$ and $1 \le x \le 1.5$ as the critical regions? 5

(c) Explain the test for the equality of means of two normal populations when population variances are unequal. 5

UNIT–V

9. (a) Define non-parametric test. When should the non-parametric method be preferably used? 2+2=4

(b) Explicate sign test for testing paired samples. 5

(c) Stating the underlying assumptions and the null hypothesis, develop the Wald-Wolfowitz run test. 5

10. (a) Explain the main difference between parametric and non-parametric approaches to the theory of statistical inference. 5

(b) Describe Wilcoxon test. 5

(c) How can one use the Kolmogorov-Smirnov test for two sample problem? 4