April 2025
B.A./B.Sc.
Sixth Semester
DISCIPLINE SPECIFIC ELECTIVE – 3
PHYSICS
Course Code: PHD 6.11
(Advanced Mathematical Physics - II)

Total Mark: 70   *Pass Mark: 28*
Time: 3 hours

Answer <u>five</u> questions, taking <u>one</u> from each unit.

UNIT–I

1. (a) Using the concept of calculus of variation, set up Euler's equation.   9
   (b) Show that the geodesic on a flat surface is a straight line.   5

2. (a) Set up Lagrange equation from D'Alembert's principle.   9
   (b) Using Lagrange equation, find the equation of motion of a simple pendulum and hence find its time period for a small amplitude oscillation.   5

UNIT–II

3. (a) Establish the relation $H = \sum_j p_j \dot{q}_j - L$ and also show that $H = T + V$, where symbols have their usual meaning.   5
   (b) Deduce Hamilton's variational principle.   5
   (c) For what values of m and n do the transformation equations $Q = q^m \cos(np), P = q^m \sin(np)$ represent a canonical transformation.   4

4. (a) Set up Hamiltonian canonical equations of motion.   6
   (b) Using the concept of generating functions, show that the transformation $P = \frac{1}{2}(p^2 + q^2), Q = \tan^{-1}\left(\frac{p}{q}\right)$ is canonical.   5

(c) If $\{p_l, q_i\}, \{q_l, q_i\}$ are Lagrange brackets and $\left[p_l, p_j \right], \left[q_l, p_j \right]$ are the Poisson brackets, then prove that

$$\sum_{l=1}^{n} \{p_l, q_i\} \left[p_l, p_j \right] + \sum_{l=1}^{n} \{q_l, q_i\} \left[q_l, p_j \right] = 0.$$ 3

UNIT–III

5. (a) Establish the relation $(ab)^{-1} = b^{-1}a^{-1} \, \forall \, a, b \in G$.   4

(b) Show that $A = \{(0,1,2,3,4,5), +_6\}$ is a cyclic group. Find all its generators.   5

(c) Workout the reducible representation of NH_3 and resolve into irreducible representation.   5

6. (a) Let G be the set of all positive rational numbers and $*$ be the operation on G defined by $a * b = \dfrac{ab}{7} \, \forall \, a, b \in G$. Prove that

$(G, *)$ is an abelian group.   5

(b) Write 3×3 matrix representation of identity operation (E) and hence find its character value.   4

(c) Find the missing (?) irreducible representation of the following point group. (Given order = 4)

	E	X_1	X_2	X_3
Γ_1	1	1	1	1
Γ_2	1	-1	1	-1
Γ_3	1	-1	-1	1
Γ_4	?	?	?	?

5

UNIT–IV

7. (a) A man is known to speak the truth 3 out of 4 times. He rolls a six-sided die and reports that the result is a six. What is the probability that the actual outcome is a six?   4

(b) A box contains 16 bulbs, out of which 4 bulbs are defective. Three bulbs are drawn at random from the box. Let X be the

number of defective bulbs drawn. Find the probability distribution of X. Also compute the mean and variance of X.   6

(c) An urn contains 8 red balls, 3 yellow balls, and 9 white balls. Six balls are randomly selected with replacement. What is the probability that exactly 2 red, 1 yellow, and 3 white balls are selected?   4

8. (a) If two television sets are randomly selected in succession without replacement from a shipment of 240 sets, of which 15 are defective, what is the probability that both selected sets will be defective?   4

(b) A random variable X has the following probability distribution

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find the value of k and calculate $P(X \geq 6)$.   5

(c) Three players A, B, and C play a series of matches. The probability that player A wins any given game is 20%, the probability that player B wins is 30%, and the probability that player C wins is 50%. If they play 6 games, what is the probability that player A wins exactly 1 game, player B wins exactly 2 games, and player C wins exactly 3 games?   5

UNIT–V

9. (a) The probability of a worker catching a cold while working in an ice factory during winter is 25%. What is the probability that more than 3 out of 5 workers will catch a cold?   4

(b) A manufacturer produces medicine bottles and finds that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles each. A drug manufacturer purchases 100 boxes from this bottle producer. Using the Poisson distribution, find the number of boxes (out of 100) that are expected to contain:
(i) No defective bottles
(ii) At least 2 defective bottles
(iii) At most 2 defective bottles
(Given: $e^{-5} = 0.6065$)   5

(c) Fit a straight line of the form $y = a\,x + b$ to the following data using the least squares method. Also, estimate the value of y when $x = 7$.

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

5

10. (a) A multiple-choice test consists of 20 questions, each with four answer choices: A, B, C, and D. Only one of the four choices is correct for each question. If a student answers all 20 questions by guessing randomly, what is the probability that they will answer exactly 6 questions correctly?   4

(b) A car rental firm has two cars available for hire each day. The number of requests for a car on any given day follows a Poisson distribution with a mean of 1.5.
   (i) What is the proportion of days on which there is no demand?
   (ii) What is the proportion of days on which demand is refused?
   (Given: $e^{-1.5} = 0.2231$)   5

(c) A set of data for the age (in months) and weight (in kg) of six babies is shown below. Fit a regression line and predict the weight of a baby at the age of 15 months.

X (Age in months)	5	4	3	7	11	12
Y (Weight in kg)	6	6	5	8	8	9

5

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