April 2025
B.A./B.Sc.
Fourth Semester
CORE – 8
PHYSICS
Course Code: PHC 4.11
(Mathematical Physics - III)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Evaluate $\frac{(\cos\theta + i\sin\theta)^8}{(\sin\theta + i\cos\theta)^4}$ in the form $(x + iy)$. 4

   (b) If ω is a cube root of unity, prove that $(1-\omega)^6 = -27$. 5

   (c) If $\alpha, \alpha^2, \alpha^3, \alpha^4$ are the roots of $x^5 - 1 = 0$ find the roots and show that $(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4) = 5$. 5

2. (a) Prove the necessary condition for the function $f(z)$ to be analytic. 5

   (b) Show that the function $e^x(\cos y + i\sin y)$ is analytic, real, and imaginary. Find its derivative. 5

   (c) Determine whether $\frac{1}{z}$ is analytic or not. 4

UNIT–II

3. (a) Evaluate $\int_{1-i}^{2+i}(2x + iy + 1)\,dz$ along the path $x = t + 1$, $y = 2t^2 - 1$. 4

   (b) Expand the function $f(z) = \frac{1}{z}$ about $z = 2$ in Taylor's series. 5

(c) Use Cauchy's integral formula to evaluate $\int \frac{z}{\left(z^2-3z+2\right)} dz$, where c is the circle $|z-2|=\frac{1}{2}$. 5

4. (a) Find the order of each pole and residue of the function $f(z)=\frac{1-2z}{z(z-1)(z-2)}$. 4

(b) Using residue theorem, evaluate $\int \frac{1+z}{z(2-z)}$, where C is the circle $|z|=1$. 5

(c) Evaluate $\int_0^{2\pi} \frac{d\theta}{5-4\sin\theta}$. 5

UNIT–III

5. (a) Express the function

$$f(x)=\begin{cases} 1 & \text{when} |x| \le 1 \\ 0 & \text{when} |x| > 1 \end{cases}$$

as a Fourier integral. Hence, evaluate $\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda} d\lambda$. 5

(b) Find the complex form of Fourier integral representation of

$$f(x)=\begin{cases} e^{-kx}, & x>0 \text{ and } k>0 \\ 0, & \text{otherwise} \end{cases}$$ 5

(c) Find the Fourier transform of $f(t)=\begin{cases} t, \text{for} |t|<a \\ 0, \text{for} |t|>a \end{cases}$ 4

6. (a) Obtain Fourier cosine transform of $f(x) = e^{-ax}$. 5

(b) State and prove the convolution theorem on Fourier transform. 5

(c) If $F_1(s)$ and $F_2(s)$ are Fourier transforms of $f_1(x)$ and $f_2(x)$ respectively, then show that,
$F\left[af_1(x)+bf_2(x)\right]=aF_1(s)+bF_2(s)$. 4

UNIT–IV

7. Find the Laplace transforms of

(a) $f(x)=\begin{cases} t-1, & 1<t<2 \\ 3-t, & 2<t<3 \end{cases}$ 5

(b) $f(x)=4\cosh 2t \sin 4t$ 5

(c) $\int_0^t \frac{\sin t}{t} dt$ by using integral theorem 4

8. (a) Compute Laplace transform of the function (half-wave rectifier)

$$f(t)=\begin{cases} \sin \omega t & \text{for } 0<t<\frac{\pi}{\omega} \\ 0 & \text{for } \frac{\pi}{\omega}<t<\frac{2\pi}{\omega} \end{cases}$$ 6

(b) Express the following function in terms of unit step function and find its Laplace transform $f(t)=\begin{cases} 0, & 0<t<1 \\ t-1, & 1<t<2 \\ 1, & 2<t \end{cases}$ 6

(c) Find the inverse Laplace transform of $\frac{1}{s^2-9}$. 2

UNIT–V

9. (a) Solve $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$ for $x \geq 0, t \geq 0$ under the given conditions $u=u_0$ at $x=0, t>0$ with initial condition $u(x, 0) = 0$, $x \geq 0$.
7

(b) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions: 7

(i) $u = 0$ when $x = 0$, $t > 0$

(ii) $u = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$ when $t = 0$

(iii) $u(x, t)$ is bounded

10. (a) An infinitely long string having one end at $x = 0$ is initially at rest along x-axis. The end $x = 0$ is given a transverse displacement $f(t)$ when $t > 0$. Find the displacement of any point of the string at any time. 7

(b) Using Laplace transforms, find the solution of the initial value problem: 7

(i) $y'' - 4y' + 4y = 64 \sin 2t$

(ii) $y'' + 9y = 6 \cos 3t$