April 2025
B.A./B.Sc.
Second Semester
MINOR – 2
MATHEMATICS
Course Code: MAN 2.11
(Calculus)

Total Mark: 50 *Pass Mark: 20*
Time: 2 hours

I. Answer five questions, taking one from each unit.

UNIT–I

1. (a) Using l'Hôpital's rule, evaluate any two of the following: 2×3=6

(i) $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$

(ii) $\lim_{x\to +\infty}\left(1-\frac{3}{x}\right)$

(iii) $\lim_{x\to 0}\frac{x-\tan^{-1}x}{x^3}$

(b) Suppose that the number of bacteria in a culture at time t is given by $N=5000\left(25+te^{-t/20}\right)$. Find the largest and the smallest number of bacteria in the culture during the time interval $0\le t\le 100$. 6

2. (a) If $\cos\left(\frac{y}{b}\right)=\log\left(\frac{x}{n}\right)^n$, prove that

$$x^2y_{n+2}+\left(2n+1\right)xy_{n+1}+\left(2n+1\right)xy_{n+1}+2n^2y_n=0.$$ 6

(b) Sketch the graph of in $r=a\sin 3\theta$ polar coordinates. 6

UNIT–II

3. (a) Derive a reduction formula for $\int x^m (\ln x)^n dx$ and hence evaluate $\int x^3 (\ln x)^2 dx$. 4

(b) The position of a particle $P(x,y)$ at time t is given by $x(t)=\frac{1}{3}(2t+3)^{3/2}, y(t)=\frac{t^2}{2}+t$. Find the distance it travels between $t=0$ and $t=3$. 4

(c) Find the volume of the solid that results when the region enclosed by $x=y^2$ and $x=y$ is revolved about the line $y=-1$. 4

4. (a) Evaluate: 2×2=4

(i) $\int \sin^4 x \cos^5 x dx$

(ii) $\int \tan^3 4x dx$

(b) Find the volume of the solid generated when the region enclosed by $y=\sqrt{x+1}, y=\sqrt{2x}$ and $y=0$ is revolved about the x-axis. 4

(c) Find the area of the surface generated by rotating about the y-axis, the arc of the curve $y=x^2$ between (0, 0) and $(2,4)$. 4

UNIT–III

5. (a) Show that $\tan^{-1} x=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), -1<x<1$ and hence find $\frac{d}{dx}\tan^{-1} x$. 2+2=4

(b) Let $\vec{r_1}(t)=(\tan^{-1} t)\hat{i}+(\sin t)j+t^2 k$ and $\vec{r_2}(t)=(t^2-t)\hat{i}+(2t-2)j+(\ln t)k$. The graphs of $\vec{r_1}$ and $\vec{r_2}$ intersects at the origin. Find the degree measure of the acute angle between the tangent lines to the graphs of $\vec{r_1}(t)$ and $\vec{r_2}(t)$ at the origin. 4

(c) Find the escape speed in km/s for a space probe in a circular orbit that is 300 km above the surface of the Earth. 4

6. (a) Evaluate: 2+2=4

(i) $\int \coth 5x dx$

(ii) $\int x \sinh x dx$

(b) Prove that $\frac{d}{dt}\left[\vec{u}(t).\vec{v}(t)\right] = \frac{d}{dt}\vec{u}(t).\vec{v}(t) + \vec{u}(t).\frac{d}{dt}\vec{v}(t)$. 4

(c) Find the elevation angles that will enable a shell, fired from ground level with muzzle speed of 800 ft/s, to hit a ground level target 10,000 ft away. 4

II. Answer <u>any two</u> *of the following questions.* 7×2=14

7. Sketch the graph of $y = \frac{(x+1)^2}{1+x^2}$.

8. Find the volume generated by revolving the triangular region bounded by $2y = x+4, y = x$ and $x = 0$ about the x-axis and also about the y-axis.

9. The position of a particle is given by $\vec{r}(t) = e^t\vec{i} + e^{-2t}\vec{i} + t\vec{k}$. Find

(i) the scalar tangential and normal components of acceleration at $t = 0$.

(ii) the vector tangential and normal components of acceleration at $t = 0$.

(iii) the curvature of the path at the point where the particle is located at $t = 0$.