April 2025
B.A./B.Sc.
Second Semester
MAJOR – 2
MATHEMATICS
Course Code: MAM 2.11
(Algebra)

Total Mark: 50 *Pass Mark: 20*
Time: 2 hours

I. Answer <u>three</u> *questions, taking* <u>one</u> *from each unit.*

UNIT–I

1. (a) Prove that $x^4 - 16x^3 + 86x^2 - 176 + 105$ is exactly divisible by $(x+4)$. 3
   (b) Solve the cubic equation $2x^3 - 21x^2 + 42x - 16 = 0$, given that the roots are in G.P. 4
   (c) Solve the biquadratic equation $x^4 - 4x^3 - 4x^2 - 24x + 15 = 0$. 5

2. (a) Find the multiple roots with multiplicity of the equation $x^3 + x^2 - 16x + 20 = 0$. 3
   (b) If α, β, γ be the roots of $x^3 + px - q = 0$, find the equation whose roots are $\alpha^2 + \beta^2,\ \beta^2 + \gamma^2,\ \gamma^2 + \alpha^2$. 4
   (c) Solve $x^3 + x^2 - 9x + 12 = 0$ by Cardan's method. 5

UNIT–II

3. (a) Prove that the matrix $\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal and write the inverse. 3
   (b) Prove that a square matrix can be uniquely expressed as a sum of symmetric matrix and skew symmetric matrix. 4

(c) Solve the given system of equations

$$x+2y+3z=6$$
$$3x-2y+z=2$$
$$4x+2y+z=7$$ 5

4. (a) Show that the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ is nilpotent and find its index. 3

(b) State and prove right distributive law of matrix multiplication with respect to addition. 4

(c) If the matrix $A=\begin{pmatrix} -1 & 2 \\ 2 & 3 \end{pmatrix}$ and $B=\begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$, verify that $(A+B)^2 = A^2+AB+BA+B^2$. Can this be put in the form $A^2+2AB+B^2$? 5

UNIT–III

5. (a) Determine the rank of the matrix $A=\begin{pmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{pmatrix}$ by reducing it to echelon form. 3

(b) Prove that every non-singular matrix is row equivalent to a unit matrix. 4

(c) Find the inverse of the matrix $A=\begin{pmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{pmatrix}$ by using elementary row transformation. 5

6. (a) Find the values of a, b, c, d in the matrix $A=\begin{pmatrix} a & b & c \\ d & e & -4 \\ 5 & -6 & 7 \end{pmatrix}$ so that $BA = AB$, where the matrix $B=\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, and then find the value of e so that the rank of the product $BA(=AB)$ is 2. 3

(b) Find the inverse of the matrix $A=\begin{pmatrix} 0 & 2 & 3 \\ 2 & 4 & 0 \\ 3 & 0 & 1 \end{pmatrix}$ by using elementary row transformation. 4

(c) Determine the rank of the matrix $A=\begin{pmatrix} 3 & -2 & 0 & 1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$ by reducing it to normal form. 5

II. Answer <u>any two</u> *of the following questions.*

7. (a) If $x=-1+\sqrt{2}$ is a root of the equation $x^4+(1-2\sqrt{2})x^3+(4-2\sqrt{2})x^2+(3-4\sqrt{2})x+1=0$, is $x=-1-\sqrt{2}$ also a root? Justify your answer. 2

(b) If α, β, γ, δ be the roots of $x^4-3x^3+4x^2-5x+6=0$, find the values of $(\alpha^2+3)(\beta^2+3)(\gamma^2+3)(\delta^2+3)$. 3

(c) Find the equation with real coefficients having $1-3i$ and 2 as two of its roots. 2

8. (a) If $AB = A$ and $BA = B$, show that A and B are idempotent. 2

(b) If A^T and B^T are the transposes of A and B then prove the relation $(A+B)^T = A^T + B^T$. 3

(c) Give an example of a complex matrix which is skew-symmetric but not skew-Hermitian. 2

9. (a) Is the following pair of matrices equivalent? 3

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}, B = \begin{pmatrix} -1 & -2 & 3 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{pmatrix}$$

(b) Under what condition is the rank of the given matrix 3? 2

$$A = \begin{pmatrix} 2 & 4 & 2 \\ 2 & 1 & 2 \\ 1 & 0 & x \end{pmatrix}$$

(c) Prove that every non-singular matrix A is expressible as the product of elementary matrices. 2
