April 2025
B.A./B.Sc.
Fourth Semester
GENERIC ELECTIVE – 4
MATHEMATICS
Course Code: MAG 4.11
(Differential Equations & Higher Trigonometry)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. Solve the following:
   (a) $x^4 \frac{dy}{dx} + x^3 = -\sec(xy)$ 5
   (b) $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$ 5
   (c) $\frac{dy}{dx} = \frac{y}{x} + x\sin\frac{y}{x}$ 4

2. Solve the following:
   (a) $(x^3 - x)\frac{dy}{dx} - (3x^2 - 1)y = x^5 - 2x^3 + x$ 5
   (b) $(2x^2 + 3y^2 - 7)dx - (3x^2 - 2y^2 - 8)ydy = 0$ 5
   (c) $(2xy + x^2)y = 3y^2 + 2xy$ 4

UNIT–II

3. Solve the following:
   (a) $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$ 5
   (b) $y = 2px + yp^2$ 5
   (c) $(x + y - 10)dx + (x - y - 2)dy = 0$ 4

4. (a) Solve $\left(2xy\cos x^2 - 2xy + 1\right)dx + \left(\sin x^2 - x^3 + 2\right)dy = 0$. 5

(b) Find the complete solution of $\left(D^3 - 3D^2 + 4D - 2\right)y = \cos x$. 5

(c) Solve $\left(y^2 - x^2\right)dx + 2xydy = 0$. 4

UNIT–III

5. Solve the following:

(a) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$ 5

(b) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 5e^{3x}$ 5

(c) $\left(D^2 + 4\right)y = \cos 2x$ 4

6. (a) Solve $\left(D^3 - 3D + 4D - 2\right)y = e^x + \cos x$. 5

(b) Solve $\left(D^2 - 4D + 4\right)y = x^3e^{2x}$. 5

(c) The equation of motion of a particle is given by $\frac{dx}{dt} + \omega y = 0, \frac{dy}{dt} - \omega x = 0$. Find the path of the particle and show that it describes a circular motion. 4

UNIT–IV

7. (a) State and prove De Moivre's theorem. 5

(b) If $2\cos\theta = x + \frac{1}{x}$ and $2\cos y = y + \frac{1}{y}$, prove that one of the values of $x^m y^n + \frac{1}{x^m y^n}$ is $2\cos\left(m\theta + n\phi\right)$. 5

(c) Show that $\cos^5\theta = \frac{1}{16}\left[\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right]$. 4

8. (a) Expand $\cos^9\theta$ in a series of cosines of multiples of θ. 5

(b) Expand $\sin^7\theta$ in a series of sines multiples of θ. 5

(c) If $\frac{\sin\theta}{\theta} = \frac{5765}{5766}$, show that $\theta = 2°$ approximately. 4

UNIT–V

9. (a) Prove that $\tanh(\alpha+\beta) = \frac{\tanh\alpha + \tanh\beta}{1+\tanh\alpha\ \tanh\beta}$. 5

(b) If $\sin(A+iB) = x+iy$, prove that $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$ and $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$. 5

(c) Prove that $\log\tan\left(\frac{\pi}{4} + i\frac{x}{2}\right) = i\tan^{-1}(\sinh x)$. 4

10. (a) Prove that if $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$, then $\theta = \tan\theta - \frac{1}{2}\tan^3\theta + \frac{1}{5}\tan 5\theta - \ldots$ 5

(b) Prove that $\sqrt{2}\left(1 - \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \ldots\right) = \frac{\pi}{4}$. 5

(c) Prove that $\frac{\pi}{4} = \left(\frac{2}{3} + \frac{1}{7}\right) - \frac{1}{3}\left(\frac{2}{3^3} + \frac{1}{7^3}\right) + \frac{1}{5}\left(\frac{2}{3^5} + \frac{1}{7^5}\right)$. 4