

**April 2025**  
**B.A./B.Sc.**  
**Sixth Semester**  
**DISCIPLINE SPECIFIC ELECTIVE – 3**  
**MATHEMATICS**  
*Course Code: MAD 6.11*  
(Theory of Equations)

Total Mark: 70  
Time: 3 hours

Pass Mark: 28

Answer five questions, taking one from each unit.

**UNIT-I**

1. (a) Without actual division, show that  $(x-1)^{2n} - x^{2n} + 2x - 1$  is divisible by  $2x^3 - 3x^2 + x$ . 3
- (b) Prove that every equation of degree  $n$  has  $n$  roots and no more. 5
- (c) (i) Find the equation whose roots are  $2 - 3i, 1 + i$ . 3
- (ii) Find the maximum and minimum values of  $f(x) = 10x^3 - 17x^2 + x + 6$  3
2. (a) Apply Descartes' rule of sign to show that the equation  $x^4 + 15x^2 + 7x - 10 = 0$  has at least two imaginary roots. 5
- (b) Find the condition that the roots  $\alpha, \beta, \gamma, \delta$  of  $x^4 + px^3 + qx^2 + rx + s = 0$  should be connected by the relation  $\alpha\beta = \gamma\delta$ . 5
- (c) Solve the equation  $x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$ , whose roots are in A.P. 4

**UNIT-II**

3. (a) If  $\alpha, \beta, \gamma, \delta$  be the roots of a biquadratic equation, then find the total number of terms in 3
- (i)  $\sum \alpha^2\beta\gamma$
- (ii)  $\sum \frac{\alpha}{\beta}$

- (b) If  $\alpha, \beta, \gamma$  are the roots of the cubic equation  $x^3 + px^2 + qx + r = 0$ , then find in terms of the coefficient the values of the following symmetric functions: 5
- (i)  $\sum \alpha^2$
- (ii)  $\sum \alpha^2 \beta$
- (iii)  $\sum \alpha^3 \beta^2$
- (c) (i) Remove the fractional coefficient of the equation  $x^3 - \frac{5}{2}x^2 - \frac{7}{18}x + \frac{1}{108} = 0$ . 3
- (ii) Find the equation whose roots are greater by 2 than the roots of the equation  $x^3 - 5x^2 + 6x - 3$ . 3
4. (a) Find the equation whose roots are the squares of the roots of the equation  $x^3 + ax^2 + bx + c = 0$ . 4
- (b) Reduce the equation  $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$  to the standard form and hence solve it. 5
- (c) If  $\alpha$  is a non-real root of  $x^7 - 1 = 0$ , find the equation whose roots are  $\alpha + \alpha^6, \alpha^2 + \alpha^5, \alpha^3 + \alpha^4$ . 5

### UNIT-III

5. (a) Solve  $x^3 - 6x - 9 = 0$  by Cardan's method. 4
- (b) Show that the roots of the equation  $x^3 - 3x^2 + 3 = 0$  are  $\left(1 - 2 \cos \frac{\pi}{9}\right), \left(1 - \cos \frac{5\pi}{9}\right), \left(1 - 2 \cos \frac{7\pi}{9}\right)$ . 5
- (c) Solve the equation  $x^4 + 8x^3 - 34x^2 - 392x - 735 = 0$ . 5
6. (a) Solve the biquadratic equation  $x^4 + 5x^3 + x^2 - 13x + 6 = 0$  by Descartes' method. 5
- (b) Solve  $x^4 + 12x = 5$  by Ferrari's method. 5
- (c) If  $\alpha_1$  be the root of the equation  $f(x) = 0$  of multiplicity  $r$ , then prove that  $\alpha_1$  is the root of the equation  $f'(x) = 0$  of multiplicity  $(r - 1)$ . 4

## UNIT-IV

7. (a) Obtain a finite superior limit of the positive roots of the equation  $x^5 + 3x^4 + 3x^3 - 28x^2 - 61x + 18 = 0$ . Also find an inferior limit of the positive roots, superior and inferior limits of the negative roots of the equation. 5
- (b) If  $\alpha, \beta, \gamma$  be the roots of the equation  $x^3 + px^2 + qx + r = 0$ , then
- (i) form an equation whose roots are  $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}, \gamma + \frac{1}{\gamma}$
- (ii) find the value of  $\left(\frac{1}{\beta} + \frac{1}{\gamma} - \frac{1}{\alpha}\right)\left(\frac{1}{\alpha} + \frac{1}{\gamma} - \frac{1}{\beta}\right)\left(\frac{1}{\alpha} + \frac{1}{\beta} - \frac{1}{\gamma}\right)$  6
- (c) Find the sum of the 4<sup>th</sup> power of the roots of  $7x^3 + 7x^2 + 1 = 0$ . 3
8. (a) Find the sum of the cubes of the roots of the equation  $x^3 - 6x^2 + 11x - 6 = 0$ . 3
- (b) Prove that every rational symmetric function of the roots of an algebraic equation can be expressed rationally in terms of the coefficients. 6
- (c) If  $\alpha, \beta, \gamma$  be the roots of  $x^3 + 2x^2 - 3x - 1 = 0$ , find the value of  $\alpha^{-5} + \beta^{-5} + \gamma^{-5}$ . 5

## UNIT-V

9. (a) Find the consecutive integers which contain the real roots of  $f(x) = x^4 - x^3 - 4x^2 + 4x + 1 = 0$ . 5
- (b) Use Sturm's theorem to show that the equation  $x^3 - 7x + 7 = 0$  has one root between  $-4$  and  $3$  and two roots between  $1$  and  $2$ . 5
- (c) Find the condition that the roots of the equation  $z^4 + 6Hz^2 + 4Gz - a^2I - 3H^2 = 0$  are real. 4
10. (a) Using Newton's theorem of approximation, find a root of the equation  $x^3 - 2x - 5 = 0$ , correct to two places of decimal. 3
- (b) Find by Horner's method, the positive roots of the equation  $x^3 - 2x^2 - 5 = 0$ , correct to four decimal places. 5

- (c) Find to 9 decimal places the root of the equation  $x^3 - 2x - 5 = 0$  which lies between 2 and 3. 6

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