April 2025
B.A./B.Sc.
Fourth Semester
CORE – 8
MATHEMATICS
Course Code: MAC 4.11
(Numerical Methods)

Total Mark: 70 *Pass Mark: 28*
Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Write an algorithm to find the real roots of the quadratic equation $ax^2\ bx+c=0$. 5
   (b) If $\Delta x = 0.005$ and $\Delta y = 0.001$ be the absolute errors in $x = 2.11$ and $y = 4.15$, find the relative error in the computation of $x+y$. 3
   (c) Using $\log\left(\frac{1+x}{1-x}\right) = 2\left(x+\frac{x^3}{3}+\frac{x^5}{5}\right)$, determine $\log_e(1.2)$ up to four places of decimal. 4
   (d) Using quadratic formula, find the smaller root of the equation $x^2 - 400x + 1 = 0$ correct to four places of decimal. 2

2. (a) Write an algorithm to find the factorial of *n* where *n* is a non-negative integer. 5
   (b) If $R = \frac{5xy^2}{z^3}$, then find the maximum relative error in *R* given that $\Delta x = \Delta y\ = \Delta z = 0.001$ and $x = y = z = 1$. 5
   (c) Determine the order of convergence for the sequence $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$, where $x_n > 0$, $a > 0$. 4

UNIT–II

3. (a) Determine the rate of convergence for the secant method. 7

(b) Perform five iterations of the Newton-Raphson method to find the smallest positive root of the equation $x^3 - 5x + 1 = 0$. 7

4. (a) Perform four iterations to find the cube root of 7 by the *regula falsi* method. 7

(b) Find a root of the equation $f(x) = x^3 - 4x - 9 = 0$ lying between 2 and 3 by bisection method. Perform four iterations. 7

UNIT–III

5. (a) Let A be a square matrix. Then, show that $\lim_{m \to \infty} A^m = 0$ if $\|A\| < 1$ or if and only if $\rho(A) < 1$. 4

(b) Perform four iterations by Gauss-Jacobi method using $X^{(0)} = [0.5,\ 0.5, -0.5]^T$:

$$4x_1 + x_2 + x_3 = 2$$
$$x_1 + 5x_2 + 2x_3 = -6$$
$$x_1 + 2x_2 + 3x_3 = -4$$
5

(c) Solve the system of equation using Gaussian elimination with partial pivoting:

$$\begin{bmatrix} 3 & 1 & 4 & -1 \\ 2 & -2 & -1 & 2 \\ 5 & 7 & 14 & -8 \\ 1 & 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 20 \\ -4 \end{bmatrix}$$
5

6. (a) Find the inverse of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ by using LU decomposition method taking $l_{ii} = 1$. 8

(b) Consider the system of equation $\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ where a is a real constant. For which value of a, the Jacobi and Gauss-Seidel iteration scheme converge? 6

UNIT–IV

7. (a) The table below gives the value of tan(x) for $0.10 \le x \le 0.30$:

x	0.10	0.15	0.20	0.25	0.30
$y = \tan(x)$	0.1003	0.1511	0.2027	0.2553	0.3093

Using Gregory-Newton forward difference interpolation formula, find the required values:

(i) tan (0.12)

(ii) tan (0.26) 3+3=6

(b) Using $\sin(0.1) = 0.099983$ and $\sin(0.2) = 0.19867$, find an approximate value of $\sin(0.15)$ by Lagrange interpolation. Obtain a bound on the truncation error. 5

(c) Prove that $\Delta\nabla = \nabla\Delta$. 3

8. (a) The population of a town was as given. Estimate the population for the year 1925: 5

Year (x)	1891	1901	1911	1921	1931
Population (y) (in-lakhs)	46	66	81	93	101

(b) Given $\log_{10} 100 = 2$, $\log_{10} 101 = 2.0043$, $\log_{10} 103 = 2.0128$, $\log_{10} 104 = 2.0170$, find $\log_{10} 102$. 5

(c) Use the Newton's divided difference formula to calculate $f(3)$ from the table: 4

(x)	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

UNIT–V

9. (a) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with the boundary condition $y = 1$ for $x = 0$. Find approximately for $x = 0.1$ by Euler's method by taking the step size 0.02. 7

(b) Evaluate the integral $\int_0^6 \frac{dx}{1+x^2}$ by trapezoidal rule by dividing the interval into six equal parts. Evaluate the same by Simpson's $\frac{1}{3}^{rd}$ rule and compare them with the original value of the integral. 7

10. (a) Given $\frac{dy}{dx} = y - x$, where $y(0) = 2$ and $h = 0.1$, find $y(0.1)$ and $y(0.2)$ correct to four decimal places using Runge-Kutta second order formula. 7

(b) Use midpoint rule with $n = 4$ to approximate the definite integral $\int_{-0.5}^{3.5} \frac{x^3}{4} dx$. 4

(c) Evaluate $\int_0^4 \frac{dx}{4x+5} dx$ by Boole's rule taking five ordinates. 3