2022

M.Sc.

Fourth Semester

DISCIPLINE SPECIFIC ELECTIVE – 04

MATHEMATICS

Course Code: MMAD 4.21

(Fluid Dynamics)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Describe the Lagrange's and Eulerian methods of describing the fluid flows. The velocity distribution of a certain 2-D flow is given by $u = Ay + b$ and $v = Ct$, where A, B, C are constants. Obtain the equation of motion of fluid particles in Lagrangian method. 4+3=7

   (b) Determine the acceleration at a point $(2,1,3)$ at $t = 0.5$ sec, if $u = yz + t, v = xz - t$ and $w = xy$. 3

   (c) Derive the equation of continuity by vector approach for incompressible fluid. 4

2. (a) Show that the surface $\frac{x^2}{a^2k^2t^4} + kt^2\left(\frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = 1$ is a possible form of boundary surface of a liquid at time t. 5

   (b) Determine the stream lines and the path lines of the particle when the components of velocity field are given by $u = \frac{x}{1+t}$, $v = \frac{y}{2+t}$ and $w = \frac{z}{3+t}$ 2+2=4

   (c) Derive Euler's dynamical equation of fluid motion in vector form. 5

UNIT–II

3. (a) If the velocity distribution of an incompressible fluid at a point (x, y, z) is given by $\left\{3xz / r^5, 3yz/ r^5, \left(kz^2 - r^2\right) / r^5\right\}$, determine the parameter k such that it is a possible fluid motion. Hence find its velocity potential. 4+3=7

(b) Two sources, each of strength m are placed at the points $(-a,0)$ and $(a,0)$ and a sink of strength $2m$ at the origin. Show that the stream lines are the curves $\left(x^2 + y^2\right)^2 = a^2\left(x^2 - y^2 + \lambda xy\right)$, where λ is a parameter. Also show that the fluid speed at any point is $\dfrac{2ma^2}{r_1 r_2 r_3}$, where r_1, r_2, r_3 are the distances of the points from the sources and the sink. 5+2=7

4. (a) Find the image of a source with regard to a circle. 7

(b) Consider a uniform flow u_0 in the positive x-direction. A cylinder of radius a is placed at the origin. Find the stream function and velocity potential. Also, find the stagnation points. 2+3+2=7

UNIT–III

5. (a) If $u = \dfrac{ax - by}{x^2 + y^2}, v = \dfrac{ax + by}{x^2 + y^2}$ and $w = 0$ then show that motion is irrotational. Find the velocity potential and pressure associated with it. 4+3=7

(b) If two vortices are of the same strength and the spin is the same in both, show that the relative streamlines are given by

$$\ln\left(r^4 + a^4 - 2a^2r^2 \cos 2\theta\right) - \frac{r^2}{2a} = \text{constant}$$

. Angle θ is measured from the join of the vortices, the origin being its middle point, $2a$ being the distance between the vortices. 7

6. (a) Find the complex potential ω and the stream function ψ due to a set of line vortices of strength k placed at points $z = \pm na \; (n = 0,1,2,3,....)$. 7

(b) Describe Karman vortex street and find velocity components at the origin. 7

UNIT–IV

7. (a) Derive Navier-Stokes equations of motion for viscous fluid. 7
   (b) Discuss generalized plane Couette flow. 7

8. (a) Discuss some limitations of Navier-Stokes equations. Derive the expression for Reynolds number. Discuss significance of this number. 2+3+2=7
   (b) Discuss Hagen-Poiseuille flow. 7

UNIT–V

9. (a) Define the following terms: 1×7=7
   (i) Isothermal process (ii) Adiabatic process
   (ii) Entropy (iv) Enthalpy
   (v) Isentropic flow process (vi) Homentropic flow process
   (vii) Progressive wave
   (b) Derive the equation of motion of a gas in an isentropic flow process. 7

10. (a) Derive an expression for the speed of sound in air.
    An aeroplane is flying at a height of 14 km, where the temperature is –45°C. The speed of the plane corresponds to M = 2. Find the speed of the plane given that $R = 287$ J/kg.K and $\gamma = 1.4$. 4+3=7
    (b) Let the Mach number of a normal shock wave in the air be 2. If the atmospheric pressure and air density are $26.5\ \text{kN/m}^2$ and $0.413\ \text{kg/m}^3$ respectively then find the Mach number, pressure, temperature and density after the shock wave. Take $R = 287$ J/kg.K and $\gamma = 1.4$ 1+2+2+2=7