2022

M.Sc.

Fourth Semester

CORE - 11

MATHEMATICS

Course Code: MMAC 4.11

(Mathematical Methods)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) State and prove the second translation or shifting property of Laplace transform. 4

(b) Evaluate the integral 4

(i) $\int_0^\infty \frac{e^{-t}-e^{-3t}}{t}\,dt$ (ii) $\int_0^\infty te^{-2t}\cos t\,dt$

(c) If $F(t)=\begin{cases} \sin t, & 0<t<\pi \\ 0, & t>\pi \end{cases}$, show that $\mathcal{L}\{F(t)\}=\frac{1+e^{-\pi s}}{s^2+1}$ 3

(d) Find the Fourier sine and cosine transform of $f(x)=\begin{cases} 1, & 0<x<\alpha \\ 0, & x>\alpha \end{cases}$ 3

2. (a) If $\mathcal{L}^{-1}\{f(s)\}=F(t)$ and $\mathcal{L}^{-1}\{g(s)\}=G(t)$ then prove that

$$\mathcal{L}^{-1}\{f(s)g(s)\}=\int_0^1 F(u)G(t-u)\,du=F*G$$ 5

(b) Solve the differential equation $Y''+tY-Y=0, Y(0)=0, Y'(0)=1$ using Laplace transform. 5

(c) Evaluate the integral $\int_0^{\infty} \frac{dx}{\left(x^2+1\right)^2}$ using Parseval's identity. 4

UNIT–II

3. (a) Verify that $\phi(x) = \frac{x}{\left(1+x^2\right)^{\frac{5}{2}}}$ is a solution of the integral equation

$$\phi(x) = \frac{3x+2x^3}{3\left(1+x^2\right)^2} - \int_0^x \frac{3x+2x^3-t}{\left(1+x^2\right)^2}\phi(x)\,dt$$ 4

(b) Form an integral equation corresponding to the differential equation $Y''+Y = \cos x$ with the initial conditions $Y(0) = Y'(0) = 0$ 4

(c) Solve the integral equation $\phi(x) = x^2 - x^4 + \int_0^x 4t\phi(t)\,dt$ by successive substitution method. 6

4. (a) Using the method of successive approximation, solve the integral equation $\phi(x) = 1 + \int_0^x (x-t)\phi(t)\,dt$. 5

(b) Solve the integral equation using Laplace transform

$$\phi(x) = e^{2x} + \int_0^x e^{t-x}\phi(t)\,dt$$ 4

(c) Solve the integral equation with degenerated kernel 5

$$\phi(x) - \lambda\int_0^{2\pi}(\sin x\cos t - \sin 2x\cos 2t + \sin 3x\cos 3t)\phi(t)\,dt = \cos x$$

UNIT–III

5. (a) Using Laplace transform, solve the integral equation

$$\phi(x) = x + \int_0^x \sin(x-t)\phi(t)\,dt.$$ 4

(b) Using Fredholm's determinant find the resolvent kernel and hence solve the integral equation $\phi(x) - \lambda\int_0^1 (2x-t)\phi(t)\,dt = 1$ 6

(c) Find the eigenvalue and eigenfunction of the integral equation

$$\phi(x) - \lambda\int_0^{2\pi} \sin x \cos t\,\phi(t)\,dt = 0$$ 4

6. (a) Find the characteristic number and eigenfunction of the homogenous integral equation $\phi(x) - \lambda\int_0^{\pi} K(x,t)\phi(t)\,dt = 0$, where

$$K(x,t) = \begin{cases} \cos x \sin t, & 0 \le x \le t \\ \cos t \sin x, & t \le x \le \pi \end{cases}$$ 7

(b) Show that the integral equation $\phi(x) - \lambda\int_0^1 \left(t\sqrt{x} - x\sqrt{t}\right)\phi(t)\,dt = 0$ does not have real characteristic number and eigenfunction. 5

(c) Define integral equation with symmetric kernel with example. 2

UNIT–IV

7. (a) Prove that convolution is associative. 4

(b) Show that $l^3\left\{n^2 \cos nt\right\} = l^2 - \left\{\frac{1}{n}\sin nt\right\}$, where l = integral operator. 4

(c) Prove that 6

(i) $\left\{\frac{1}{\beta}e^{\alpha t}\sin\beta t\right\} = \frac{1}{(s-\alpha)^2 + \beta^2}$

(ii) $\left\{e^{\alpha t}\cos\beta t\right\} = \dfrac{s-\alpha}{(s-\alpha)^2+\beta^2}$, where s = differential operator.

8. (a) Derive the relation between Dirac delta function and Heaviside unit function. 4

(b) Show that $\lim\limits_{a\to\infty} F(a,x) = \lim\limits_{a\to\infty} \dfrac{1}{\pi}\dfrac{a}{x^2+a^2}$ is a Dirac delta function. 4

(c) Prove that $\delta\left(x^2-a^2\right) = \dfrac{1}{2|a|}\left[\delta(x-a)+\delta(x+a)\right], a \neq 0$ 3

(d) Evaluate $\int\limits_{-\infty}^{\infty} \delta(a-x)\delta(x-b)\,dx$ 3

UNIT–V

9. (a) Convert the differential equation $x^3y''-xy'+2y=0$ to a Sturm-Liouville form. 2

(b) Find the eigenvalue and function of the Sturm-Liouville problem

$y''+\lambda y=0,\ 0<x<L,\ y(0)=0, hy(L)+y'(L)=0, h>0$ 5

(c) Express the function $f(x)=x$ as the eigenfunction series of the given Sturm-Liouville problem.

$y''+\lambda y=0,\ 0<x<1,\ y(0)=0, y(1)+y'(1)=0$ 7

10. (a) Reduce the boundary value problem to an integral equation using Green's function $y''+\lambda y=x^2,\ 0<x<\dfrac{\pi}{2},\ y(0)=y\left(\dfrac{\pi}{2}\right)=0$ 7

(b) Using Green's function solve the boundary value problem

$y''-y=0,\ 0<x<1,\ y(0)=y(1)=0$ 7