2022

M.Sc.

Second Semester

CORE – 05

MATHEMATICS

Course Code: MMAC 2.11

(Partial Differential Equations)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Discuss, in detail, the compatible systems of first order partial differential equations 9

   (b) Find the general integral of the linear PDE: $\frac{y^2 z}{x} p + xzq = y^2$ 5

2. (a) Find the characteristics of the partial differential equation $pq = xy$ and determine the integral surface which passes through the curve $z = x, y = 0$ 5

   (b) Find the complete integral of the partial differential equation

   $\left(p^2 + q^2\right)x = pz$ where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. 4

   (c) Solve: $\left(1 + x^2\right)\frac{dy}{dx} + 2xy - 4x^2 = 0$ 5

UNIT–II

3. (a) Classify and reduce the partial differential equation

$y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y$ to canonical form and hence solve it. 5

(b) Determine the adjoint operator L^* corresponding to

$L(u) = Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu$,where A, B, C, D, E and F are functions of x and y only 4

(c) Find the families of characteristics of the partial differential equation

$(1-x^2)u_{xx} - u_{yy} = 0$ in the elliptic and hyperbolic cases. 5

4. (a) Find the complete solution of the partial differential equation

$(D^2 + 3DD' + 2D'^2)u = x + y$ 5

(b) Solve, by Monge's method, the partial differential equation

$z(qs - pt) = pq^2$ 6

(c) Prove that, if $a_iD + b_iD' + c_i$ is a factor of $F(D, D')$ and $\phi_i(\xi)$ is an arbitrary function of a single variable ξ and if $a_i \neq 0$, then

$u_i = exp\left(-\frac{c_i}{a_i}x\right)\phi_i(b_ix - a_iy)$ is a solution of the equation

$F(D, D')u = 0$. 3

UNIT–III

5. (a) Discuss in detail, the interior Dirichlet problem for a circle. 9

(b) State and prove the mean value theorem for harmonic functions. 5

6. (a) Using the method of variables separable, discuss the solution of Laplace equation in cylindrical coordinates. 7
   (b) Discuss the solution of interior Neumann problem for a circle. 7

UNIT–IV

7. (a) State the elementary solution of the diffusion equation and hence verify it. 3
   (b) Define Dirac-Delta function. 3
   (c) Discuss the solution of the diffusion equation in cylindrical coordinates using variables separable method. 8

8. (a) State and prove the maximum-minimum principle in the case of diffusion equation. 7
   (b) Discuss the solution of Burger's equation in one-dimension. 7

UNIT–V

9. (a) Discuss, in detail, the D'Alembert's solution of the one-dimensional wave equation. 7
   (b) Discuss the solution of the problem of vibrating string using variables separable method. 7

10. (a) Discuss the problem of vibration of a circular membrane. 8
    (b) State and prove the Duhamel's principle. 6