2022

B.A./B.Sc.

Sixth Semester

DISCIPLINE SPECIFIC ELECTIVE – 3

PHYSICS

Course Code: PHD 6.11

(Advanced Mathematical Physics – II)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Derive the necessary conditions for Legendre transformation. 4

(b) Use Lagrange's equations to find the equation of motion of a compound pendulum in a vertical plane about a fixed horizontal axis. Hence, find the period of small amplitude oscillations of the compound pendulum. 5

(c) A surface is generated by revolving a curve $y(x)$ around the x-axis. Apply variational principle to the curve $y(x)$ which passes through two fixed points (x_1, y_1) and (x_2, y_2) so that the area is minimum. 5

2. (a) Prove that, if f does not depend on x explicitly, then

$$f - f'\frac{\partial f}{\partial y} = \text{ constant} .$$ 4

(b) Deduce general form of Lagrange's equation using D'Alembert's principle. 10

UNIT–II

3. (a) The Hamiltonian of a particle is given by $H=\sqrt{p^2+m^2}+v(x)$, where p is the momentum, m is the mass and $v(x)$ is the potential of the system. Find the Lagrangian of the system. 4

   (b) Prove that $\left[\vec{P},\vec{L},\vec{n}\right]=\vec{n}\times\vec{P}$, $\vec{P}=$ linear momentum, $\vec{L}=$ angular momentum and take $\vec{n}=n_x\hat{i}+n_y\hat{j}+n_z\hat{k}$. 5

   (c) Consider a simple pendulum having a mass 'm' is attached to a string of length 'l'. Let the length of the string be shortened at a constant rate $\frac{dl}{dt}=-\alpha$, where α is a constant. After the pendulum is set into motion, obtain the Hamiltonian and verify if the total energy is conserved. 5

4. (a) Show that the transformation $q=\sqrt{2P}\sin Q$ and $p=\sqrt{2P}\cos Q$ is canonical. 4

   (b) Prove that for any three functions F, G and K of p_k and q_k, the following holds true: $\left[F,\left[G,K\right]\right]+\left[G,\left[K,F\right]\right]+\left[K,\left[F,G\right]\right]=0$ 5

   (c) Using Hamilton's equations of motion, show that the Hamiltonian $H=\frac{p^2}{2m}e^{-rt}+\frac{1}{2}m\omega^2x^2e^{rt}$ leads to the equation of motion of a damped harmonic oscillator $\ddot{x}+r\dot{x}+\omega^2x=0$. 5

UNIT–III

5. (a) Show that the cube root of unity is an Abelian under multiplication. 6

   (b) If $a,b\in G$, then show that the equation $a*x=b$ and $y*a=b$ have unique solution in the group G. 8

6. (a) Find all the generators of the cyclic group $\left\{a,a^2,a^3,a^4,a^5,a^6,a^7,a^8=e\right\}$. 6

(b) Verify the group property of symmetry of an equilateral triangle (D_3), also known as C_{3V} 8

UNIT–IV

7. (a) Two dice are thrown simultaneously and the sum of the numbers obtained is found to be 7. What is the probability that the number 3 has appeared at least once? 4

(b) A coin is tossed three times. Find the probability of getting head one time and also the probability of getting tail three times. 5

(c) Find the constant c such that the function $f(x)=\begin{cases} cx^2, & 0<x<2 \\ 0, & \text{otherwise} \end{cases}$

is a density function and compute $P(1<x<2)$. 5

8. (a) A, B and C companies produce 25%, 35% and 40% bulbs respectively out of which 5%, 4% and 2% are defective. A bulb is drawn at random and found to be defective. What is the probability that it is from company B. 4

(b) Let X be a continuous random variable with probability distribution

function $f(x)=\begin{cases} 2x^{-2} & \text{for } 1<x<2 \\ 0 & \text{otherwise} \end{cases}$

Find E(X) and variance. 5

(c) A urn contains 4 white and 6 red balls. Four balls are drawn at random from the urn. Find the probability distribution of the number of white balls. 5

UNIT–V

9. (a) Fit a straight line to the following data with x as independent variable. 4

x	0	1	2	3	4
y	–1	–0.2	1.3	2.5	4.4

(b) Show that the normal distribution is the limiting case of a binomial distribution. 10

10. (a) Show that the mean deviation from mean of the normal distribution is about $\frac{1}{5}$ of its standard deviation. 4

(b) Find the first four moments of binomial distribution. 10