2022

B.A./B.Sc.

Sixth Semester

DISCIPLINE SPECIFIC ELECTIVE – 3

MATHEMATICS

Course Code: MAD 6.11

(Theory of Equations)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) State and prove the remainder theorem. 1+3=4

   (b) Express 2½×2=5

   (i) $x^3 - 10x^2 + 3x + 3$ as a polynomial in $(x-3)$

   (ii) $2x^4 - x^3 - 2x^2 + 5x - 1$ as a polynomial in $(x+3)$

   (c) Show that the equation $\frac{a^2}{x-a'} + \frac{b^2}{x-b'} + \cdots + \frac{k^2}{x-k'} = x - m$, where $a, b, ..., k, m, a', b', ..., k'$ are all real numbers, cannot have a non real roots. 5

2. (a) Discuss the nature of the roots of the equation 2×2=4

   (i) $x^5 + x^3 - 2x^2 + x - 2 = 0$

   (ii) $x^4 + 15x^2 + 7x - 10 = 0$

   (b) Solve $x^3 - 9x^2 + 14x + 24 = 0$ given that two of the roots are in the ratio 3:1. 5

   (c) If the roots of $x^3 + 3px^2 + 3qx + r = 0$ are in harmonic progression then show that $2q^3 = r(3pq - r)$. 5

UNIT – II

3. (a) Transform the equation $\frac{2}{3}x^4 - \frac{1}{3}x^3 + \frac{1}{16}x - \frac{1}{64} = 0$ into another with integral coefficient and its leading coefficient unity. 2

   (b) Find the polynomial whose roots are the cubes of the roots of the equation $x^3 + 33x^2 + 12x + 8 = 0$. 6

   (c) Derive the equation whose roots are the square difference of the roots of the equation $x^3 + 6x^2 + 9x + 4 = 0$ and hence solve it. 6

4. (a) Solve the equation $x^5 + 6x^4 - 7x^3 - 7x^2 + 6x + 1 = 0$. 5

   (b) Find the special roots of the equation $x^8 - 1 = 0$. 4

   (c) If α is a non-real root of $x^7 - 1 = 0$, find the equation whose roots are $\alpha + \alpha^6, \alpha^2 + \alpha^5, \alpha^3 + \alpha^4$. 5

UNIT – III

5. (a) Using Cardan's method, find the roots of the equation $2x^3 + 3x^2 + 3x + 1 = 0$. 5

   (b) Determine the roots of the equation $x^3 - 12x + 8 = 0$. 4

   (c) Solve $x^4 + 5x^3 + x^2 - 13x + 6 = 0$. 5

6. (a) Solve $x^4 - 12x^3 + 41x^2 - 18x = 72$ using Ferrari's method. 5

   (b) Show that the equation $x^4 + 3x^3 - 7x^2 - 15x + 18 = 0$ has multiple roots. 4

   (c) Find the Euler's cubic of the equation $x^4 + 8x^3 - 34x^2 - 392x - 735 = 0$ and hence solve it. 5

UNIT – IV

7. (a) Define symmetric function of the roots of an equation. 1

   (b) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the following values in terms of p, q and r 2×4=8

(i) $\sum \alpha^3$ (ii) $\sum \alpha^2\beta^2$

(iii) $\sum \frac{1}{\alpha}$ (iv) $\sum \frac{1}{\alpha^2}$

(c) Using Newton's theorem find the values of $\sum \alpha^6$ and $\sum \alpha^7$, where α,β,γ,and δ are the roots of the equation $x^4 + 6x^2 + 9x - 1 = 0$. 5

8. (a) Find the limits of the roots of the equation $x^3 - 9x^2 + 13x - 23 = 0$. 7

(b) Show that –8 is a superior limits of the negative roots of the equation $x^4 - 2x^3 - 13x^2 + 38x - 24 = 0$. 4

(c) Form an equation whose roots are $\beta\gamma + \frac{1}{\alpha}, \gamma\alpha + \frac{1}{\beta}$ & $\alpha\beta + \frac{1}{\gamma}$, where α,β,γ are the roots of the equation $x^3 - px^2 + qx - r = 0$. 3

UNIT – V

9. (a) Using theorem of Fourier & Budan and DeGau's rule for imaginary roots, find the character of the roots of the equation $x^8 + 10x^4 + x - 4 = 0$. 5

(b) Find all the Sturm's function and the position of the real roots of the equation $x^5 + 2x^4 + x^3 - 4x^2 - 3x - 5 = 0$. 5

(c) Find the roots of the equation $x^3 + x^2 + x = 100$ correct to three decimal places using Newton's method of approximation. 4

10. (a) Using Horner's method, find a positive roots of the equation $x^3 - 3x + 1 = 0$ correct to seven decimal places. 7

(b) Find, correct to four decimal places, the value of the two nearly equal roots of the equation $x^3 - 7x + 7 = 0$ which lie between 1 & 2. 7