2022

B.A/B.Sc.

Fourth Semester

GENERIC ELECTIVE – 4

MATHEMATICS

Course Code: MAG 4.11

(Differential Equation & Higher Trigonometry)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five questions, taking one from each unit.

UNIT–I

1. (a) Fine the equation of the curve represented by $(y-yx)dx+(x+xy)dy=0$ and passing through the point $(1,1)$ 3

(b) Solve: $\dfrac{dy}{dx}=\dfrac{x+2y-3}{2x+y-3}$ 6

(c) Solve the Bernoulli's equation $(x^3y^2+xy)dx=dy$ 5

2. (a) Solve $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+10y+37\sin 3x=0$, and find the value of y when $x=\dfrac{\pi}{2}$ if it is given that $y=3$ and $\dfrac{dy}{dx}=0$ when $x=0$ 6

(b) Solve: $\dfrac{d^2y}{dx^2}-3\dfrac{dy}{dx}+2y=e^x$ 4

(c) Solve: $x^2\dfrac{d^2y}{dx^2}+y=3x^2$ 4

UNIT – II

3. (a) Solve: $y(1+xy)dx + x(1-xy)dy = 0$ 4

(b) Solve: $p^3 + (2x - y^2)p^2 - 2xy^2 p = 0$ 5

(c) Solve the Lagrange's equation $y = 3px + 4p^2$ 5

4. (a) Solve: $(y^2 + 2x^2 y)dx + (2x^3 - xy)dy = 0$ 5

(b) Solve: $yp^2 - 2xp + y = 0$ 5

(c) Find the general and singular solution of $y = px + \frac{a}{p}$ 4

UNIT – III

5. (a) Solve by the method of variation of parameters

$$x^2 \frac{d^2 y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(x+1)y = x^3$$ 6

(b) Solve: $\frac{d^2 y}{dx^2} - 2\tan x \frac{dy}{dx} + 5y = 0$ 4

(c) Solve the ordinary simultaneous equations

$$\left(\frac{d}{dt} + 2\right)x + 3y = 0$$

$$3x + \left(\frac{d}{dt} + 2\right)y = 2e^{3t}$$ 4

6. (a) Solve: $2(y+z)dx - (x+z)dy + (2y - x + z)dz = 0$ 5

(b) Solve the simultaneous equation $\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{zxy - 2x^2}$ 4

(c) Solve: $xz^3 dx - 3dy + 2ydz = 0$ 5

UNIT – IV

7. (a) Find the equation whose roots are $\sec^2\frac{2\pi}{7}, \sec^2\frac{4\pi}{7}, \sec^2\frac{6\pi}{7}$

Also find the value of $\sec\frac{2\pi}{7}+\sec\frac{4\pi}{7}+\sec\frac{6\pi}{7}$ 5

(b) Prove that $(1+i)^n+(1-i)^n = 2^{\frac{n}{2}+1}\cos\left(\frac{n\pi}{4}\right)$ 4

(c) Prove that $\cos\frac{2\pi}{7}, \cos\frac{4\pi}{7}, \cos\frac{6\pi}{7}$ are the roots of the equation

$8x^3+4x^2-4x-1=0$ 5

8. (a) If θ be small, prove that $\theta\cot\theta = 1-\frac{\theta^2}{3}-\frac{\theta^4}{45}$ approximately. 4

(b) Show that $\frac{\pi^2}{2\cdot 4}-\frac{\pi^4}{2\cdot 4\cdot 6\cdot 8}+\frac{\pi^6}{2\cdot 4\cdot 6\cdot 8\cdot 10\cdot 12}-....=1$ 4

(c) Expand $\sin^7\theta$ in a series of sines of multiple of θ 6

UNIT – V

9. (a) Prove that $\sin^{-1}(\operatorname{cosec}\theta) = \left\{2n+(-1)^n\right\}\frac{\pi}{2}+i(-1)^n\log\cot\frac{\theta}{2}$

5

(b) Prove that $\frac{\pi}{8}=\frac{1}{1\cdot 3}+\frac{1}{5\cdot 7}+\frac{1}{9\cdot 11}+\cdots\infty$. 4

(c) Sum the series $\sin\alpha+\sin(\alpha+\beta)+\sin(\alpha+2\beta)+\cdots$ to n terms.

5

10. (a) Use the Euler's exponential values to prove that

$\cos x-\cos y = 2\sin\frac{1}{2}(x+y)\sin\frac{1}{2}(y-x)$ 4

(b) Resolve $\tanh(\alpha + i\beta)$ into real and imaginary parts. 5

(c) Prove that $\log\left(\frac{1}{1-e^{i\alpha}}\right) = \log\left(\frac{1}{2}\operatorname{cosec}\frac{\alpha}{2}\right) + i\left(\frac{\pi}{2} - \frac{\alpha}{2}\right)$ 5
