2022

B.A./B.Sc.

Fourth Semester

CORE – 9

STATISTICS

Course Code: STC 4.21

(Linear Models)

Total Mark: 70 *Pass Mark: 28*

Time: 3 hours

Answer five *questions, taking* one *from each unit.*

UNIT–I

1. (a) Define Gauss-Markov linear model in terms of matrix notations. When is the model called an analysis of variance model, regression model and analysis of covariance model? 4+3=7

   (b) Define estimable function. Prove that a necessary and sufficient condition for the linear function $\tilde{l}^T\tilde{\beta}$ of the parameters to be linearly estimable is $Rank(A) = Rank\begin{pmatrix} A \\ \tilde{l}^T \end{pmatrix}$, where $\begin{pmatrix} A \\ \tilde{l}^T \end{pmatrix}$ is the matrix defined from A adjoining the row vector $\tilde{l}^T$. 2+5=7

2. (a) What is a BLUE? Define parametric function. 2+3=5

   (b) If $\tilde{l}^T$ is any estimable linear function with parameters $\beta_1, \beta_2, ..., \beta_p$, then prove that

   (i) $\exists$ a unique linear function $\tilde{C}^T\tilde{Y}$ of the random variables $Y_1, Y_2, ..., Y_n$ such that $\tilde{C} \in V(A^T)$ and $E(\tilde{C}^T\tilde{Y}) = \tilde{l}^T\tilde{\beta}$. 4

   (ii) $Var(\tilde{C}^T\tilde{Y})$ is less than the variance of any other linear unbiased estimator of $\tilde{l}^T\tilde{\beta}$. 3

(c) Show that the best estimator of any estimable function $\tilde{l}^T \tilde{\beta}$ must be of the form $\tilde{q}^T A^T \tilde{Y}$, where $\tilde{q}^T = (q_1, q_2, ..., q_p)$ is a row vector and satisfies the equation $\tilde{q}^T A^T A = \tilde{l}^T$. 2

UNIT–II

3. (a) Describe the test for the linearity of regression using the technique of analysis of variance. 7

(b) How is the analysis of variance technique used to test for the homogeneity of a group of regression coefficients? 7

4. (a) Write a note on using the analysis of variance technique in the test for polynomial regression. 7

(b) Delineate the test for equality of regression equations from p groups using the analysis of variance technique. 7

UNIT–III

5. (a) Describe what you mean by analysis of variance technique. State the assumptions in analysis of variance test. 5+2=7

(b) Represent the yield of a plot of a one-way layout with one observation per cell for analysis of variance by the fixed effect linear model stating the meaning and assumptions of the notations used and also estimate the parameters involved in it. 3+4=7

6. (a) In the analysis of one-way classified data, the following information has been given in the ANOVA table

Sources of variation	d.f.	SS	MS	F
Between classes	4	…	…	2.5
Within class	…	…	20	
Total	19	500		

Find the missing values. 4

(b) Suppose in a two-way classified data with 5 classes and 4 groups, the observed difference between two classes is 10 and error mean square (EMS) is 20. Given that $t_{0.025,12} = 2.18$ then calculate the critical difference (CD) and comment if the observed difference between two classes be statistically significant. 4

(c) In analysis of variance for one-way classification of data, show that the expectation of between class mean square is given by

$E\left(\frac{S_C^2}{k-1}\right) = \sigma_e^2 + \frac{1}{k-1}\sum_{i=1}^{k} n_i\alpha_i^2$, where notations have their usual meanings. 6

UNIT–IV

7. (a) What is analysis of covariance? Illustrate with examples. 4+2=6

(b) In analysis of covariance for one-way classified data with one concomitant variable, state the fixed effect linear model and obtain the estimated values of the parameters involved in it. 8

8. (a) What is a concomitant variable? Give two examples. 4

(b) Give the layout of the analysis of covariance of a two-way classified data with one concomitant variable. 3

(c) Write down the analysis of covariance table in usual notation for one-way classified data with one concomitant variable. 4

(d) Write down the analysis of covariance linear model for the data in two-way classification with one concomitant variable stating the meaning of each notation used in it. 3

UNIT–V

9. (a) Define Marginal Propensity to Consume. John's income in the month of March was ₹10,000 and his consumption for the month of March was ₹5,000. But in the month of April his income increases to ₹12,000 and his consumption also increases to ₹6,000. How much percent of the increased income did he use for the purpose of consumption? 2+2=4

(b) Explain the specification of the mathematical model of consumption. 4

(c) If the inter correlation between the explanatory variables is perfect $\left(r_{x_i x_j} = 1\right)$, prove that the estimates of the coefficients are indeterminate. 6

10. (a) Define: 4

(i) Multicollinearity

(ii) Autocorrelation

(b) Explain the sources of autocorrelation. 4

(c) Find the mean, variance and covariance of autocorrelation. 6