**2022**

**B.A./B.Sc.**

**Fourth Semester**

CORE – 8

**STATISTICS**

*Course Code: STC 4.11*

(Statistical Inference)

*Total Mark: 70* *Pass Mark: 28*

*Time: 3 hours*

*Answer* five *questions, taking* one *from each unit.*

## UNIT–I

1. (a) Distinguish between: 2×2=4
   (i) Parameter and statistic
   (ii) Estimator and estimate

   (b) Define consistent estimator. Let a random sample of size $n$ be drawn from a normal population $N(\mu, \sigma^2)$. Prove that the sample mean is a consistent estimator of population mean. 6

   (c) If $T_1$ and $T_2$ are two unbiased estimators of $\gamma(\theta)$, having the same variance and $\rho$ is the correlation coefficient between them, then show that $\rho \geq 2e-1$, where $e$ is the efficiency of each estimator. 4

2. (a) Define an estimator. What are the properties of a good estimator? 2+2=4

   (b) State the sufficient conditions for consistency of an estimator. 2

   (c) Define an unbiased estimator. 2

   (d) A random sample $(X_1, X_2, X_3, X_4, X_5)$ is drawn from a normal population with unknown mean $\mu$. Consider the following estimators to estimate $\mu$:

   (i) $t_1 = \frac{1}{5}(X_1 + X_2 + X_3 + X_4 + X_5)$

(ii) $t_2 = \frac{1}{2}(X_1 + X_2) + X_3$

(iii) $t_3 = \frac{1}{3}(2X_1 + X_2 + \lambda X_3)$

Where $\lambda$ is such that $t_3$ is an unbiased estimator of $\mu$. Find the value of $\lambda$. Are $t_1$ and $t_2$ unbiased? State giving reasons, the estimator which is best among $t_1$, $t_2$ and $t_3$. 2+2+2=6

## UNIT–II

3. (a) If T is an unbiased estimator for $\gamma(\theta)$ then prove that

$$Var(T) \geq \frac{\left\{\frac{d}{d\theta}\gamma(\theta)\right\}^2}{E\left\{\frac{\partial}{\partial\theta} Logf(X_1, X_2, \ldots\ldots\ldots, X_n)\right\}^2}$$ 7

(b) A sample of 15 persons who died due to COVID-19 from the total deaths in Nagaland has been taken. The age at death has been recorded as 42, 67, 85, 55, 70, 46, 82, 51, 77, 91, 65, 68, 85, 38 and 66. Find the 95% and 99% confidence intervals for the mean age at death of all the persons who died due to COVID-19 in Nagaland. Also, comment which one of the intervals is more likely to include the true value of the population mean? 7

4. (a) State and prove the Rao-Blackwell theorem. 7

(b) Obtain the minimum variance bound estimator for $\mu$ in the normal population $N(\mu, \sigma^2)$, where $\sigma^2$ is known. Also, obtain the variance of the estimator. 6+1=7

## UNIT–III

5. (a) What is method of maximum likelihood estimation? Write down the properties of maximum likelihood estimator. 3+3=6

   (b) Obtain the maximum likelihood estimator of $\mu$ and $\sigma^2$ simultaneously on drawing a random sample from a normal population having mean $\mu$ and variance $\sigma^2$. 4

   (c) Briefly explain Baye's estimator. 4

6. (a) For the double Poisson distribution:

$$p(x) = P(X = x) = \frac{1}{2}.\frac{e^{-m_1} m_1^x}{x!} + \frac{1}{2}.\frac{e^{-m_2} m_2^x}{x!}; \quad x = 0,1,2,......., $$

   show that the estimates for $m_1$ and $m_2$ by the method of moments are: $\mu_1^{/} \pm \sqrt{\mu_2^{/} - \mu_1^{/} - \mu_1^{/2}}$ 7

   (b) Explain the method of minimum chi-square. 7

## UNIT–IV

7. (a) Suppose getting of a multiple of 3 in a single throw of a die is a success and $p$ be the probability of getting success in a single throw in order to test the $H_0 : p = \frac{1}{3}$ against $H_1 : p = \frac{2}{3}$. The die is thrown 5 times and $H_0$ is rejected if more than 3 successes are obtained. Find probability of type-I error and the power of test. 6

   (b) Derive the likelihood ratio test for testing the variance of a normal population. 8

8. (a) Define uniformly most powerful test. 2

   (b) If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $H_1 : \theta = 1$, on the basis of the single observation from the population $f(x,\theta) = \theta \exp(-\theta x); 0 \le x \le \infty$, obtain the values of type-I and type-II error. 4

(c) State and prove Neyman-Pearson lemma. 8

## UNIT–V

9. (a) Define a non-parametric test and state some of its advantages. 4

(b) Explicate Mann-Whiteney U-test for testing the identicalness of two populations. 5

(c) Describe briefly Kolmogorov-Smirnov test of goodness of fit in case of one sample. 5

10. (a) Differentiate parametric and non-parametric test. 4

(b) Describe sign test for two sample. 5

(c) Explicate Kruskal-Wallis test stating its assumptions. 5